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TRITIATED HEAVY WATER TRANSPORTATION PACKAGE: ANALYTICAL ASSESSMENT OF FLUID-STRUCTURE INTERACTION EFFECTS DURING IMPACT

OHRD Report No. 88-208-K

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ABSTRACT

Packages used in the road transportation of low level radioactive wastes such as tritiated heavy water must demonstrate the ability to withstand severe impacts such as those prescribed by the AECB regulations. One such package is the Tritiated Heavy Water Transportation Package (THWTP). Previous analyses included the liquid payload as a non-structural lumped mass, thus ignoring fluid-structure interaction (FSI). In order to provide a quantitative assessment of FSI, an analytical model was developed to simulate the hydrodynamic effects of the heavy water during a 9 meter flat end drop of the THWTP.

The results indicate that the liquid payload will have no deleterious effect on the structural integrity of the THWTP. The report presents the detailed development and formulation of the analytical model, computer code implementation, results of the simulation, and conclusions.

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EXECUTIVE SUMMARY

TRITIATED HEAVY WATER TRANSPORTATION PACKAGE: ANALYTICAL ASSESSMENT OF FLUID-STRUCTURE INTERACTION EFFECTS DURING IMPACT

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Packages used for the road transportation of low level radioactive wastes, such as tritiated heavy water, must demonstrate the ability to withstand severe impacts such as those prescribed by the AECB regulations. Currently, the Nuclear Materials Management Department (NMMD) at Ontario Hydro are in the process of designing, building and licensing the Tritiated Heavy Water Transportation Package (THWTP). Previous analyses included the heavy water payload as a non-structural lumped mass, thus ignoring possible fluid-structure interaction (FSI). At the request of NMMD, a detailed analytical assessment of the THWTP including the hydrodynamic effect of the liquid payload was carried out. The study consisted of two phases:

- i) Developing an analytical model to simulate the hydrodynamic effects of the heavy water payload during impact.
- ii) Performing an impact analysis for a 9 m flat end drop of the THWTP including fluid structure interaction.

To this end, a subset of the three dimensional Navier-Stokes momentum equations were used to develop a hydrodynamic model capable of representing a fluid body constrained by a structure.

The fluid model was incorporated into the Ontario Hydro version of the non-linear transient dynamic computer code DYNA3D. The simulation was performed on the THWTP including the hydrodynamic effects. The results indicate that the fluid response is attenuated to a large extent by the flexibility of the fluid container shell and subsequently by the combined effects of the container and the overpack when they come into contact later in

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the transient. Thus, the fluid acoustic effects on the structural components are mitigated and the fluid response is essentially governed by the deceleration of impact. The material response in this impact scenario is well below the allowable values thus precluding any damage which would compromise the structural integrity of the THWTP.

The work described in this report represents a significant enhancement of OHR capabilities in the area of structural impact problems by providing a method of dealing with FSI in complex shock problems. This report presents the detailed development and formulation of the hydrodynamic model, computer code implementation, the description of the THWTP simulation with results and conclusions.

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TRITIATED HEAVY WATER TRANSPORTATION PACKAGE: ANALYTICAL ASSESSMENT OF FLUID-STRUCTURE INTERACTION EFFECTS DURING IMPACT

1.0 INTRODUCTION

Packages used for transporting low level radioactive waste by road must demonstrate the ability to withstand severe accidental impacts such as those prescribed by the Atomic Energy Control Board (AECB) regulations/1/. The Nuclear Materials Management Department (NMMD) at Ontario Hydro are in the process of designing, building and licensing the Tritiated Heavy Water Transportation Package (THWTP). The work described herein is supplemental to the analysis of the THWTP previously carried out/6,7/. In that analysis, the heavy water payload was included as a non-structural mass. At the request of NMMD/2/, a detailed investigation of the THWTP including the fluid-structure interaction (FSI) of the liquid payload was carried out. The 9 m flat end drop was chosen as the worst scenario since this orientation results in the highest deceleration of the payload/6/.

The study consisted of two phases:

- i) Developing an analytical model to simulate the hydrodynamic effects of the heavy water payload during impact.
- ii) Performing an impact analysis for a 9 m flat end drop of the THWTP including fluid structure interaction.

The package, shown in figure 1.1, comprises an inner container filled with heavy water, and a polyurethane foam-filled overpack. The overpack is of sandwich construction with 304L stainless steel liners and 10.5 inch thick nominal foam walls. The outer shell is 0.75 inch thick and the inner shell is 0.25 inch thick. The primary containment boundary consists of the overpack inner liner, the containment lid and outer containment seals in the lid region. The design analyzed is based on the OH design drawings listed in Table 1.1.

The objective of this work is to provide information regarding the behaviour of the fluid during impact, in order to assess its effect on the structural integrity of the THWTP.

This report presents the method of analysis, the formulation of the hydrodynamic model, the description of the computer models used, general assumptions, analytical results and conclusions.

2.0 METHODS OF ANALYSIS

2.1 Mathematical Approach

The non-linear transient dynamic analysis of impacting structures involves both non-linear geometric and material behaviour affecting wave propagation in two and three-dimensional continua. A two material structure such as the THWTP, where steel and polyurethane foam interface, further complicates the problem. In addition, the consideration of the fluid-structure interaction (FSI) of the liquid payload with the containing structure adds another difficult facet to the problem. Due to the inherent complexity of the problem, the finite element method was applied using the latest Ontario Hydro version of the computer code DYNA3D./3/

The DYNA3D code is an explicit three-dimensional finite element computer program for analyzing the large deformation dynamic response of inelastic solids. A contact algorithm permits gaps and sliding along zone boundaries and material interfaces. These interfaces can be tied to admit variable zoning without the need of transition regions. Spatial discretization is achieved by the use of four, six or eight-noded isoparametric solid elements, four-noded thin shell elements and two-noded beam elements. A number of material models and equations of state are available to cover a broad range of material behaviour including elastic-plastic, soil, crushable foam, and explosive-structure interaction. The equations of motion are numerically integrated using the central difference operator (Appendix A2). This scheme is explicit and as a result, it is conditionally stable with respect to time step size. A continuous monitor of the time step is made throughout the solution to ensure that it is below a critical time step which is required to maintain operator stability. Further details of the code along with verification cases, user manual and theoretical documentation are given in references 3 and 4.

The material model constitutive relations used in DYNA3D are specified in terms of true stress and true strain. This aspect is discussed in Section 2.2. The foam and rigid portions of the package were modelled using eight node hexahedron linear isoparametric elements whereas the inner and outer stainless steel shells close to the impact zone were modelled using the Belytschko-Lin-Tsay/3/ quadrilateral thin shell element formulation.

In order to demonstrate the applicability of the computer code to the class of problem considered herein, a correlation with experimental results was carried out in references 3, 6, 7, and 8. Comparison of these results with those obtained using DYNA3D shows excellent agreement between analytical predictions and experimental results.

A simplified flow chart of the computational sequence is given in figure 2.1.1. Initially the data defining the analytical model geometry, material, integration parameters and output controls is set up. The program is executed and database files for data processing are generated. Computer programs for processing the data in terms of structural deformed plots at selected times (PLOTD), displacement/velocity time history plots and stress/strain measures for comparison with acceptance criteria (POSTD) have been developed to interface with the DYNA3D computer code. Details of the pre- and post-processors are given in reference /3/. To increase efficiency and to enable results at intermediate simulation times to be sampled for trends, a restart capability is available.

A monitoring routine has been written for VAX computers which permits DYNA3D batch runs to be interrogated for current levels of X, Y and Z direction momentum, kinetic energy, strain energy, current solution time and time step. This allows the status and convergence of the solution to be periodically inspected.

The loading on the models was obtained in terms of an initial velocity applied to the complete region defining the package, using the following relation from rigid body mechanics:

$$V_o = [2gh]^{1/2}$$

where: V_o - initial velocity at impact (in/sec)
 h - drop height (in)
 g - gravitational constant (386.4 in/sec/sec)

For $h = 354.33$ in. (9 m) $V_o = 523.3$ in/sec

2.2 Solid Material Models

For the severe impact scenario addressed in this analysis, the solid material models used, must be capable of providing an adequate representation of the moderately large strain regime. This entails a non-linear description of the material behaviour in the post-yield region in terms of:

- (a) A yield criterion to determine the threshold at which plastic flow begins;
- (b) A flow rule to relate plastic strain increments with total strain and stress increments;
- (c) A hardening rule to specify the modification of the yield condition during plastic flow; and
- (d) In the case of the polyurethane foam, an equation-of-state for pressure versus volumetric strain with compaction.

The input specification of the material constitutive relations are given in terms of logarithmic strain versus stress. Further information regarding various stress and strain measures for path dependent material properties used in large deformation analyses is given in references /3-5, 9, 11/.

The structure of the THWIP consists of two basic material types:

- (a) 304L stainless steel
- (b) 19 lb/ft³ polyurethane foam.

The parameters used to idealize each of these materials are developed and given in the following sections.

2.2.1 Stainless Steel Model

The behaviour of this ductile material is characterized by the curves shown in figure 2.2.1, depending on the stress-strain measure used. These curves are representative of the typical behaviour of a ductile material loaded first in uniaxial tension followed by uniaxial compression. The straight-line approximations serve to idealize this behaviour. A von Mises yield criterion is adopted with an isotropic hardening rule. Referring to figure 2.2.1, the straight line approximation used in the computer models is characterized by an elastic modulus E , a yield stress of σ_y , a strain hardening (plastic modulus) E_t , and a hardening parameter $\beta = 1$ for isotropic hardening.

The material properties for 304L stainless steel used in the models were taken to be:

density	-	0.29 lbf/in ³
E	-	modulus of elasticity - 28×10^6 psi
σ_y	-	yield stress - 44,300 psi
μ	-	Poisson's ratio - 0.28
K	-	$\frac{E}{3(1-2\mu)}$ - Bulk Modulus - 21.2×10^6 psi
G	-	$\frac{E}{2(1+\mu)}$ - Shear Modulus - 10.9×10^6 psi
E_t	-	strain hardening modulus
	-	2.2×10^5 psi (true stress-strain)

The idealized stress-strain curves corresponding to the various stress-strain measures used to represent the 304L SS are shown in figure 2.2.4. The data was obtained from reference /9/.

This isotropic elastoplastic material model corresponds to material type 3 in DYNA3D/3/.

2.2.2 Polyurethane Foam Model

The behaviour of the polyurethane foam material is shown schematically in figures 2.2.2 and 2.2.3 for various volumetric strain measures. Polyurethane foams exhibit a pressure dependent behaviour. They crush and compact under pressure due to the presence of voids. The model used is based on references 13 and 7, which describe a simple but useful model for soils and crushable foams whose material properties are not easily characterized. An elementary isotropic plasticity theory is utilized in which a pressure dependent flow rule governs the deviatoric behaviour, and the yield function is defined as:

$$\phi_s = J_2 - (a_0 + a_1\sigma_p + a_2\sigma_p^2)$$

where J_2 is the second invariant of the deviatoric stress tensor, σ_p is the hydrostatic pressure and a_0 , a_1 , and a_2 are user supplied constants. The pressure versus yield curve can then be described as shown in figure 2.2.3. The development of the expressions for a_0 , a_1 and a_2 are given in Appendix A3.

The value of effective yield stress is determined using $(a_0 + a_1\sigma_p + a_2\sigma_p^2)$ based on hydrostatic pressure, σ_p , and if exceeded remains at this value for subsequent pressures.

The pressure versus volumetric strain behaviour is shown in figure 2.2.2 for various strain measures. Elastic unloading is assumed to a tensile cutoff pressure using an unloading bulk modulus as shown in figure 2.2.2.

If failure under hydrostatic tension occurs, pressure is left at the cutoff value and the deviatoric stress components are zeroed. Note that pressure is positive in compression.

The data used to determine the constants for the polyurethane foam material model is based on the experimental results obtained in reference 14. A series of compressive tests on foam samples of progressively greater diameters was carried out in order to determine the minimum sample size whose in-plane expansion would have a negligible effect on its compressive stiffness. The resulting state of strain was uniaxial. The final load-displacement diagram was then equivalent to the load-volumetric change diagram. This final sample was then used to determine the engineering volumetric strain versus pressure curve and the unloading bulk modulus K_0 . The samples used were taken from the actual foam material to be used in the THWTP. The data required for input in DYNA3D consisted of a volumetric strain versus pressure curve for an equation of state with compaction in (figure 2.2.2), constants a_0 , a_1 and a_2 for the pressure dependent yield surface (figure 2.2.3), an unloading bulk modulus K_0 , shear modulus G , and pressure cutoff σ_{po} .

In reference 14, a compressive engineering volumetric strain versus pressure curve was obtained as shown in figure 2.2.6. This curve was digitized and converted to true volumetric strain versus pressure for DYNA3D as indicated in

figure 2.2.2. The remaining values of G , σ_{po} and K_o were obtained directly from reference 14. A parabolic pressure dependent flow rule as described in Appendix A3 was assumed (i.e. $a_2 = 0$).

The properties for the 19 lb/ft³ polyurethane foam used in the analysis are:

density - 0.01099 lbf/in³

Volumetric strain versus pressure, shown in figure 2.2.6, curves are based on reference /14/.

G - shear modulus = 9369.44 psi

σ_{po} - cutoff pressure = -223.84 psi (negative in tension)

K_o - $1. \times 10^6$ psi (during unloading)

Data for deriving constants a_o , a_1 as indicated in Appendix A3:

σ_{po} - cutoff pressure = -223.84 psi

σ_{yo} - 0. (yield corresponding to σ_{po})

σ_{pl} - 0. (at origin)

σ_{yl} - $\sqrt{3} \tau_{yl}$

τ_{yl} - 149.4 psi (shear yield)

Using this data and the equations in Appendix A3:

a_o - 22317.4

a_1 - 99.7

Thus, the equivalent uniaxial (effective) yield stress σ_y is given in terms of hydrostatic pressure as:

$$\sigma_y^2 = 3 [22317.4 + 99.7 \sigma_p]$$

The curve of pressure versus yield used in the analysis is given in figure 2.2.5.

In order to provide assurance that the numerical model described above accurately predicts the response of the polyurethane foam, experimental benchmarks were conducted in references /6, 7, 8/. Excellent correlation with experiment was observed. Further details are given in Appendix A3.

The crushable foam material model corresponds to material type 10 with equation of state type 8 in DYNA3D (reference 3).

2.3 General Assumption

The following assumptions were made to facilitate the analysis:

- (a) In the 9 m drop the THWTP is assumed to impact an unyielding surface. This is a conservative assumption since in reality no surface is unyielding. Even small plastic distortions of the target surface result in reduced strains (reference 10).
- (b) A bottom down flat end drop is considered since it will result in the highest deceleration of the fluid leading to the largest potential hydrodynamic pressures. As a result, the bottom portion of the package is modelled accurately with the lid assembly portion included using a rigid body representation consisting of elements of modified density (figure 5.1.2).
- (c) Due to the symmetry of this drop scenario, only one quarter of the package is modelled.

The weight distribution is taken as:

Overpack:	20,800 lbs (.75 in. outer shell)
Inner container:	3,872 lbs
Payload:	11,965 lbs
Total THWTP weight:	36,637 lbs

- (d) Strain rate sensitivity is ignored for the 304L SS material. For the range of impact velocities considered in this analysis, this assumption is justified/9/.
- (e) The analysis was discontinued once the initial impact energy and momentum was dissipated.
- (f) The liquid D₂O payload is considered inviscid (Appendix A1).
- (g) Fluid body force terms are neglected since they are small by comparison to dynamic forces (Appendix A1).
- (h) The D₂O container is considered to be fully filled with D₂O so that no air voids exist. This is conservative since this permits reflection of the initial shock wave whereas in the actual case, an air gap exists.

3.0 FLUID MODEL

3.1 Hydrodynamic Model Formulation

In order to simulate the interaction of liquid D₂O payload with the THWTP structure, a mathematical characterization of the liquid is necessary.

The basic approach taken in developing the hydrodynamic model is the formulation of a three dimensional hexahedron isoparametric element (figure 3.3) in terms associated with a fluid continuum, and incorporating it into the DYNA3D computer code/3/. The primary variables of velocities are obtained at the nodes with the secondary quantities of hydrostatic pressure sampled at the element centroid. An inviscid fluid model with an isotropic stress state is considered. A barotropic equation of state (i.e. independent of temperature) which relates volumetric strain to pressure is used to characterize the fluid behaviour. The formulation considers the fluid as a compressible media only, so that no tension fields are permitted. A subset of the three dimensional Navier-Stokes momentum equations/5/ are used to develop the model. The Galerkin weighted residual method is applied to the differential equations resulting in a discretized approximation of the continuum.

In order to verify the formulation, a one dimensional isoparametric element is derived in closed form and implemented into a one dimensional computer test code. The verification test consisted of a rigid pipe filled with water travelling at an initial velocity subjected to an instantaneous valve closure at one end. The central difference operator is used to solve the equilibrium equations in time.

Good correlation of the displacement time history and shock wave shape between the 1-D code, DYNA3D, and classical results is achieved as seen in figures 3.2 and 3.1, respectively.

Details of the mathematical derivation, verification case, and implementation into DYNA3D are given in Appendix A1.

3.2 D₂O Fluid Model

As described in the previous section and Appendix A1, the fluid behaviour is governed by an equation of state relating volumetric strain to pressure. Additional non-linearities are introduced by excluding any tension fields. This is achieved using a tension cut-off of zero pressure. The properties describing the fluid are:

density	-	0.0361 lbf/in ³
Bulk Modulus	-	300,000 lb/in ²
Cut-off pressure	-	0.0

The bulk modulus for the fluid is a function of temperature and pressure. For the range of values considered for the THWTP, the bulk modulus cited above represents an average with error bounds of approximately 3 percent which is quite acceptable.

4.0 ACCEPTANCE CRITERIA

In order to evaluate the THWTP design, some form of acceptance criteria is necessary to provide measures with which to compare the analytical results. The NUPACK committee formed by the American Society of Mechanical Engineers (ASME) to establish guidelines for the design of irradiated fuel transportation casks, have recently proposed stress and strain limiting criteria to be applied to the results of a structural analysis of impact. These criteria are intended to augment the basic stress and strain limits under level D loading conditions as given in Appendix F, Section III of the ASME code.

Appendix F does not address the problem of ductile tearing in the regions of high strains such as those existing at points of impact. This consideration is not required on account of the low values of general strain permitted under Appendix F. In addition, the hydrostatic stresses at the point of impact are usually compressive resulting in increased ductility and in a reduced tendency to crack initiation. Consequently, ductile tearing does not have to be considered in the impact region for cases involving a corner drop, edge drop or flat end drop. Thus, if hydrostatic stresses are compressive, strain limits are not imposed.

A different situation, however, may arise at the point of impact on a pin, as in a punch drop, where the hydrostatic stresses may be either tensile or compressive, and high shear stresses may cause ductile tearing.

In reference 12, two strain limitations are suggested to protect against ductile tearing (when hydrostatic stresses are tensile):

(a) For membrane strains $\epsilon_m \leq 0.7 \epsilon_n$

(b) For peak strains ϵ_{pk} :

$$\epsilon_{pk} \leq 0.7 \max \left[\left[\frac{\sinh \left[\frac{\sqrt{3}}{3} (1-n) \right]}{\sinh \left[\frac{\sqrt{3}}{3} (1-n)(TF) \right]} \right], (0.2) \right] \bar{\epsilon}_f$$

where $\bar{\epsilon}_f$ - true strain at fracture in uniaxial tension

TF - triaxiality factor = $\left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{\sigma_f} \right)$

$\bar{\sigma}_f$ - effective von Mises stress

n - strain hardening exponent in equation $\bar{\sigma} = A (B + \bar{\epsilon})^n$,

which approximates the true stress-strain curve
(Reference 11) (i.e. n = 0.5 for stainless steel,
n = 0.2 for carbon steel)

$$Z = \frac{\epsilon_{\max}}{n}$$

Values of maximum strain at failure ($\epsilon_{\max}$) are readily available for some simple cases:

- | | |
|--------------------------|-----------------------------|
| (a) Uniaxial tension | $\epsilon_{\max} = n$ |
| (b) Pressurized cylinder | $\epsilon_{\max} = 0.577 n$ |
| (c) Pressurized sphere | $\epsilon_{\max} = 0.677 n$ |

For other geometries and loading conditions the effective strain must be derived from the plastic instability condition ($dP/d\epsilon = 0$). This may present substantial difficulty when dealing with complex strain distributions or it may necessitate some conservative evaluation of the allowable effective strain.

Constants A and B in equation $\bar{\sigma} = A (B + \bar{\epsilon})^n$ can be obtained from the true stress-strain diagram. Chapter 10 of Reference 11 gives this equation for some commonly used materials.

Using the above approach, allowables for peak and membrane strains for the THWTP stainless steel shells, when the hydrostatic stresses are tensile, are obtained assuming a predominant biaxial stress distribution as follows:

$$\text{allowable membrane strain} = 0.7 \times .577 \times .5 = 0.20 \text{ (20\%)}$$

$$\text{allowable peak strain} = 0.7 \times \epsilon/\epsilon_f \times 0.5$$

From Reference 12 a value for ϵ/ϵ_f in a biaxial stress state is $\epsilon/\epsilon_f = 0.665$. Thus, allowable peak strain = $0.7 \times \epsilon/\epsilon_f \times 0.5 = 0.23$ (23%).

Note that these units only apply to stress states where the hydrostatic component is in tension and true stress-strain properties are used.

Since the outer package is not a primary containment boundary, the full absorbing capability of the structure is taken so that the allowable membrane strain = $0.20/0.7 = 0.28$ (28 percent), and the allowable peak strain = $0.23/0.7 = 0.33$ (33 percent).

In order to apply these limits some interpretation of the analytical results is necessary. In the work described herein, the effective plastic strains and effective (von Mises) stresses in the stainless steel are obtained across

selected sections of the finite element model. The resulting average and peak strains are obtained for elements whose hydrostatic stress component is tensile, and then compared to the allowable strain limits.

For cases, involving localized impact, (i.e. punch drop), the average normal shear stress across sections adjacent to the impact locations are evaluated and compared to material allowables for ASME level D faulted conditions. These are obtained from the ASME code Section III subsection NB Class 1 components as $0.6 S_u$ since the overpack is not a primary containment boundary. S_u is the ultimate strength for type 304 stainless steel. This is obtained from the ASME code Appendix F as a minimum value of 70 ksi which results in a limit on the average normal shear stress across a section of 42 ksi. In actual fact, this limit is conservative since the ultimate strength observed from data in reference 9 and the benchmark verification described in references 6 and 7 indicates that the ultimate value is more likely to be in the range of 100 ksi which would lead to an allowable in punching shear of 60 ksi.

5.0 9M BOTTOM DOWN FLAT END DROP

5.1 General Description

5.1.1 Mathematical Model

The overall model for the drop simulation consists of the package assembly and the target. The overall drop configuration is shown in figure 5.1.1. The geometric nature of the package and drop orientation permits symmetry to be exploited in order to reduce model complexity. As a result, one quarter of the package is modelled as shown in figure 5.1.2 with appropriate symmetrical boundary conditions on the XZ and YZ planes.

The package assembly is modelled from several component models which are shown in figures 5.1.2 to 5.1.7. They are identified as follows:

- overpack bottom portion - outer shell
 - inner shell
 - foam
- D_2O container
- D_2O liquid payload
- overpack upper lid portion

In the sections which follow, the development of the component models is described.

5.1.2 Mesh Generation

The generation of the mesh was initiated by setting up the bottom surface grid topology consisting of node and element numbers (figures 5.1.10 and 5.1.11) and extruding it to obtain much of the remaining mesh. The inner D_2O container and fluid mesh were generated in a similar fashion using the topology of the appropriate bottom surface shown in figures 5.1.12 and 5.1.13.

The mesh was developed to provide the refinement needed in the vicinity of the point of initial impact (i.e., $Z = 0$) while allowing a progressively coarser mesh in areas remote from the impacting surface.

The overpack body consists of inner and outer stainless steel shells with polyurethane foam between the shells. The portion of the overpack excluding the lid area is modelled using 4-noded thin shell quadrilateral elements for the stainless steel inner and outer liners, and 8-noded hexahedron elements for the foam. The shell and solid elements share common nodes where they join. The top lid portion of the overpack body is modelled using 8-noded solid elements with equivalent rigid material properties.

The bottom portion model was generated from the bottom surface upwards by replicating layers of nodes so that the numbering order is always consistent with the base as shown in figures 5.1.10 and 5.1.11.

The topology of the overpack shells and foam are given in figures 5.1.3, 5.1.5 and 5.1.7.

The top lid portion of the overpack body (figure 5.1.2 and 5.1.7) is modelled as solid 8-noded brick elements with rigid material properties of equivalent density. This part of the model is considered insignificant so far as impact effects are concerned. It was modelled to simulate the mass and inertial effects of the remaining package.

The payload, namely the D_2O container and the liquid D_2O , is modelled using 4-noded thin shell elements and the 8-noded three dimensional hexahedron fluid elements described in Section 3.0 and Appendix A1. The mesh of the inner container shell is shown in figure 5.1.4, while the mesh representing the fluid is given in figure 5.1.6. As in the case of the overpack mesh, the inner container and fluid mesh were generated from the bottom inner container surface upwards by replicating layers of nodes so that the numbering order is always consistent with the base as shown in figures 5.1.12 to 5.1.13. The model consists of 1670 node points with 1098 solids representing the polyurethane foam, fluid, rigid body portion of the model and the unyielding target. In addition, 486 shell elements represent the stainless steel D_2O inner container and the inner/outer liners of the overpack.

The clearance between the inner D_2O container and the inner liner of the overpack (figures 5.1.1 and 5.1.2) is accounted for by master/slave surface definitions as shown in figures 5.1.8 and 5.1.9.

The inner surface of the overpack defines the master surface which limits the potential motion of the inner container. The outer surface of the D_2O container defines the nodes subject to this limiting condition. This is the sliding and void feature of DYNA3D.

5.1.3 Target Model

The target model topology is shown in figures 5.1.14 and 5.1.15. In accordance with the AECB regulations, it is modelled using a rigid unyielding material. The bottom surface of the target is fully constrained in the Z-direction. As with the package, a quarter model is specified with symmetric boundary conditions on the YZ and XZ planes.

5.1.4 Modified Density

The THWTP model was developed with separate material definitions for each of the components as shown in figure 5.1.2. For all the components, except the lid region of the overpack, the materials were modelled as described in Section 2.2.

In the region of the lid an equivalent density was derived for the top layer of elements as shown in figure 5.1.2. The detailed calculations are given in Appendix A5.

The total weight of the analytical quarter model of the THWTP was 9366 lbs. This corresponds to a full model weight of 37,464 lbs. The actual total weight of the prototype package is 36,637 lbs, which is a difference of 2 percent.

5.2 Analytical Results

The 9 m bottom down flat end drop was run to 6 ms, at which time the initial drop energy was dissipated. This is apparent from the time histories of kinetic energy and impact forces shown in figures 5.2.4 to 5.2.6. As observed in the structural plots, shown in figures 5.2.1 to 5.2.3 at the sampled times of 2, 3 and 5 ms, considerable interaction of the D₂O payload, inner container and the inner liner of the overpack occur.

Examination of the respective time history plots of displacement (figures 5.2.7 to 5.2.14) and velocity (figures 5.2.15 to 5.2.22) indicates that the D₂O container and the inner liner of the overpack initially come into contact at 2.6 ms. Full spread of contact is developed at 3.5 ms at which time all components of the THWTP are loaded and the fluid hydrostatic pressure is no longer localized. Earlier in the transient the peak forces developed are due in part to the thicker outer shell and overpack body coming to rest before the payload. At various subsequent stages of the transient, the average package decelerations range from 342 g at 1.9 ms to 139 g at 5 ms when the complete package is decelerating.

These values are obtained from the drop direction impact forces for the quarter model given in figure 5.2.5.

For example, at 1.9 ms the impact force developed between the package and target is 3.2×10^6 lbs for the quarter full scale model in the drop direction. This corresponds to a deceleration $\ddot{z}$ of the total package of:

$$\bar{z} = \frac{3.2 \times 10^6}{9366} = 342 \text{ g}$$

As seen in the time histories of hydrostatic pressure in figures 5.2.23 to 5.2.25, the fluid initially responds acoustically until the first shock wave reaches the D₂O container wall at 0.6 ms.

At this point the fluid response is attenuated by the flexibility of the container shell and subsequently by the combined effects of the container and the overpack when they impact later in the transient.

The evolution of the hydrostatic pressure distribution in the fluid and the polyurethane foam during the transient is given in the fringe plots shown in figures 5.2.26 to 5.2.30. Essentially the fluid response is governed by the deceleration. The distribution in the hydrostatic pressure of the fluid is mainly due to the variation of flexibility of the inner container as it interacts with the overpack. The unloading at the end of the initial impact sequence is evident from figures 5.2.26 to 5.2.30. A peak localized hydrostatic pressure of 1100 psi occurs at the bottom of the inner container early in the transient and reaches a quasi-steady value of 450 psi in the later stages. The distribution vertically (Z-dir) is basically triangular as expected (figure 5.2.28). No surface tension effects are included so that when the fluid complies with the deformed container the fluid elements at the highest level are in a zero hydrostatic pressure state.

The time history plots of effective plastic strain for selected elements are given in figures 5.2.31 to 5.2.34. In figures 5.2.35 and 5.2.36, fringe plots of the maximum effective plastic strain distribution are shown. As observed, a maximum plastic strain of 5.3 percent occurs in the junction of the 1.00" thick flat head and the 0.25" cylindrical shell of the container. A maximum plastic strain of 2.2 percent occurs at the 0.25" inner liner of the overpack at the location of initial contact of the container and inner liner of the overpack. These levels are below the material allowables and for this scenario indicate that the containment boundary stays intact. It should be noted that welds in this region should be inspected so as to preclude any deleterious effects on the transition joints of the container from 1.00" to 0.25" thick walls.

In conclusion, the material response for this impact scenario is well below the allowable values thus precluding any damage which would compromise the structural integrity of the THWTP.

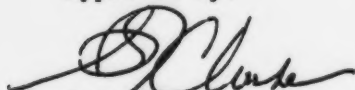
6.0 RETRIEVAL OF COMPUTER FILES

This analysis was carried out using the VAX computer facility in the Applied Mechanics Section of the Mechanical Research Department at Ontario Hydro. The computer files related to this project have been stored on one DEC TK50 tape in VMS BACKUP format. The tape is labelled RGS. A log of the backup procedure is given in Appendix A4.

7.0 ACKNOWLEDGMENTS

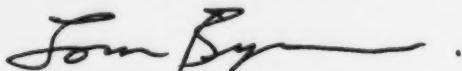
The authors would like to extend thanks to Dr. T.P. Byrne for his useful comments and suggestions and to Ms. C. Mandarino for her diligence in typing this report.

Approved by:



G.J. Clarke
Manager
Mechanical Research Department

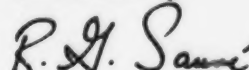
Verified (by review) by:



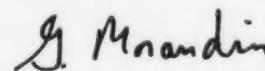
T.P. Byrne
Unit Head-Stress Analysis & Heat Transfer
Applied Mechanics Section

RGS/GDM:cam

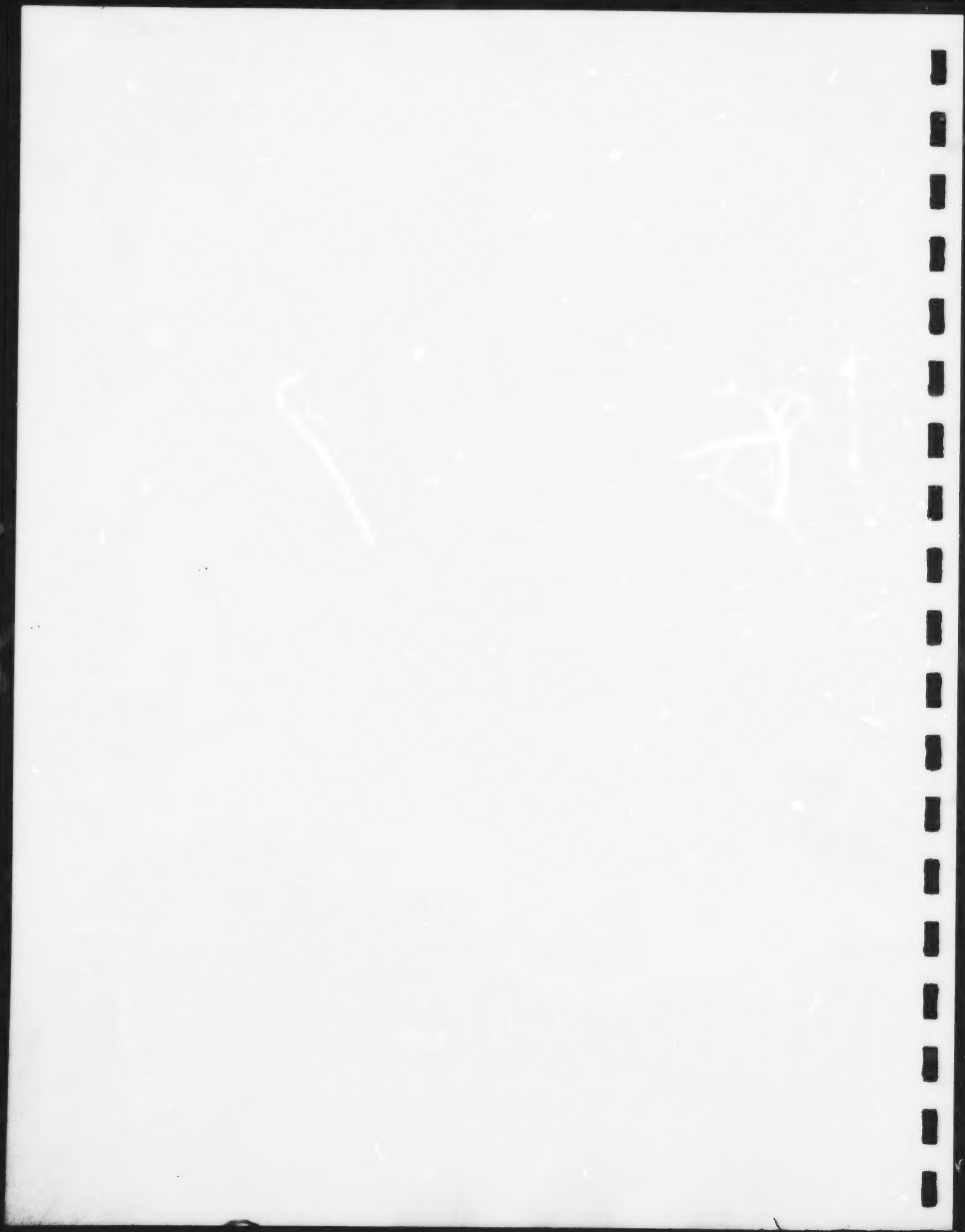
Submitted by:



R.G. Sauvé
Engineer
Applied Mechanics Section



G.D. Morandin
Technologist
Applied Mechanics Section



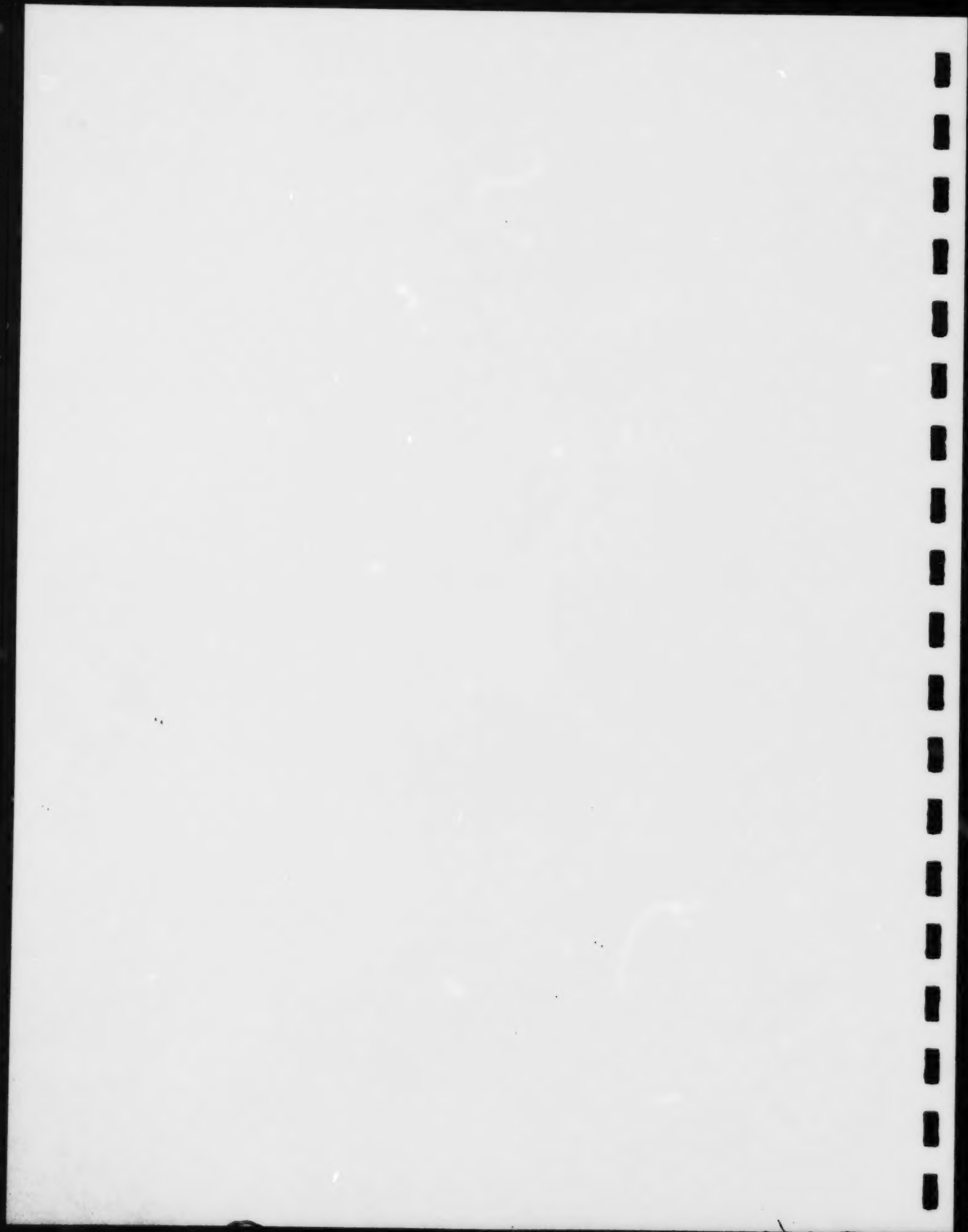
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14. Black, D.A., Mechanical Properties of Polyurethane Foam for Tritiated Heavy Water Transportation Package, OH Research Report B84-99-H, November 1984.

TABLE 1.1
LIST OF THWTP DESIGN DRAWINGS

<u>Drawing No.</u>	<u>Description</u>
NK-38-DOH-79641-0001 Rev. 4	Tritiated D ₂ O Transportation Package Assembly and Details
NK-38-DOH-79641-0002 Rev. 4	Tritiated D ₂ O Transportation Package Assembly and Details
NK-38-DOH-79641-0003 Rev. 4	Tritiated D ₂ O Transportation Package Assembly and Details
NK-38-DOH-79641-0004 Rev. 4	Tritiated D ₂ O Transportation Package Assembly and Details



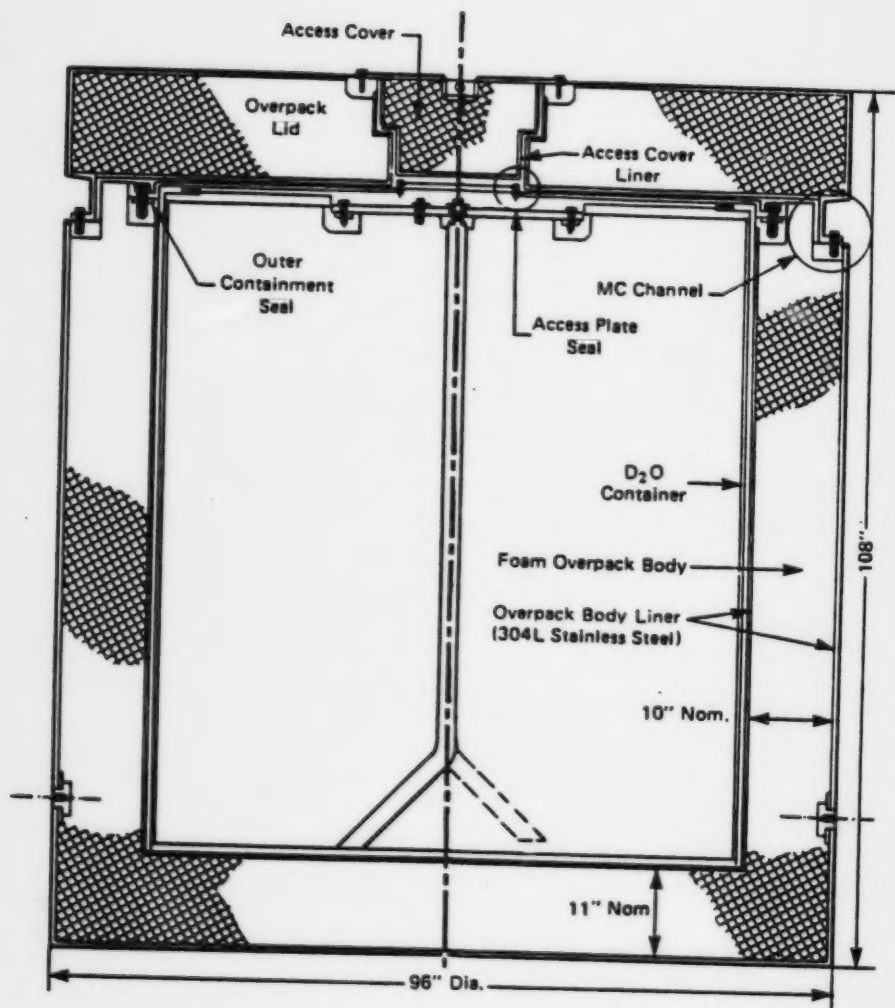


FIGURE 1.1
Tritiated Heavy Water Transportation Package

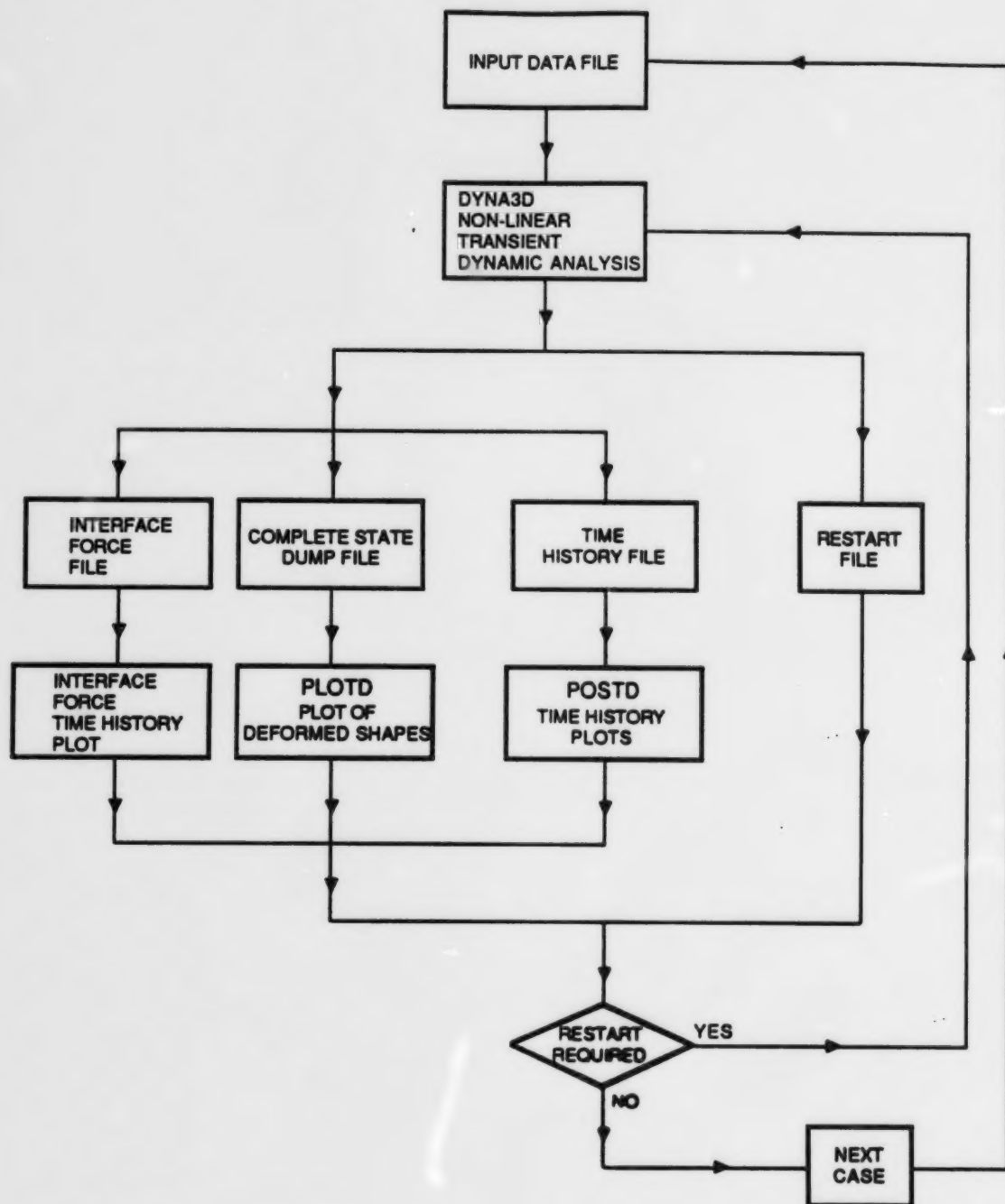


FIGURE 2.1.1
FLOW CHART OF COMPUTATIONAL SEQUENCE

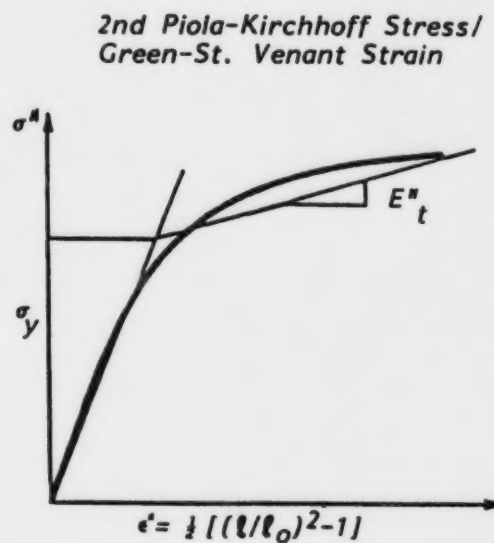
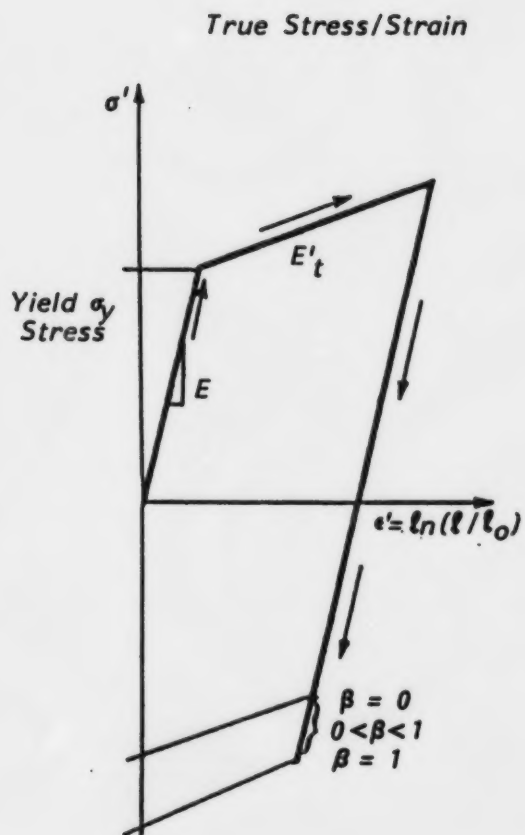
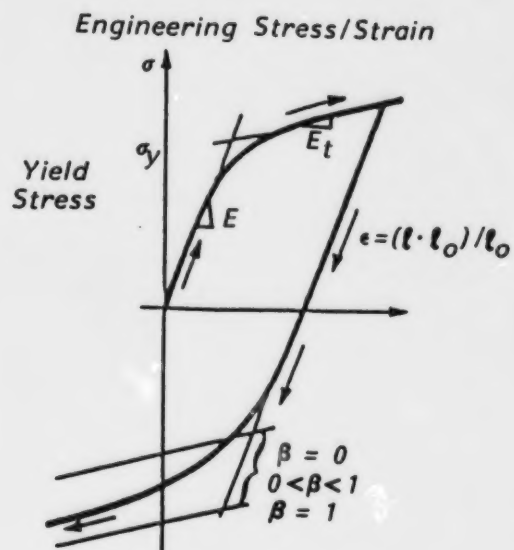
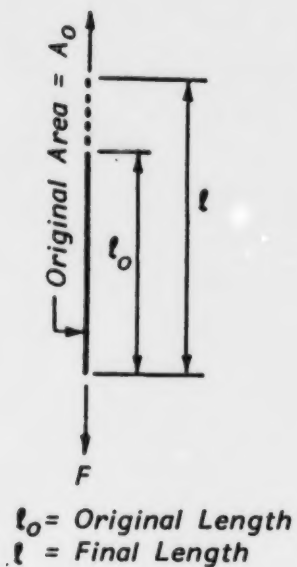


FIGURE 2.2.1
 STRESS/STRAIN MEASURES
 ELASTO-PLASTIC MATERIAL MODELS

Engineering Pressure/Volumetric Strain

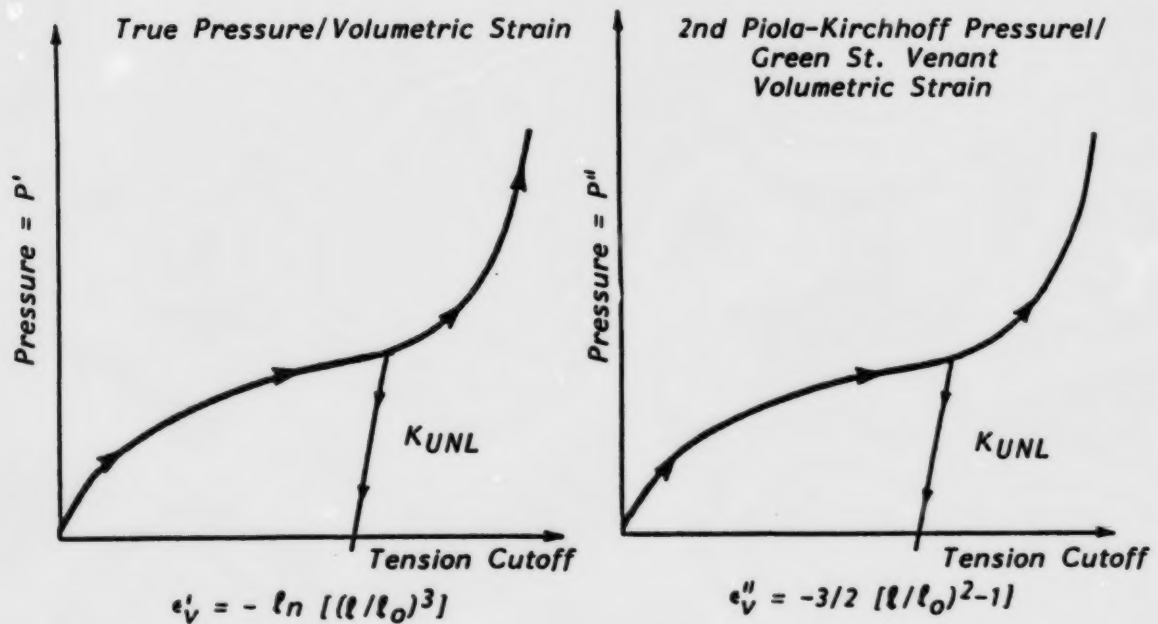
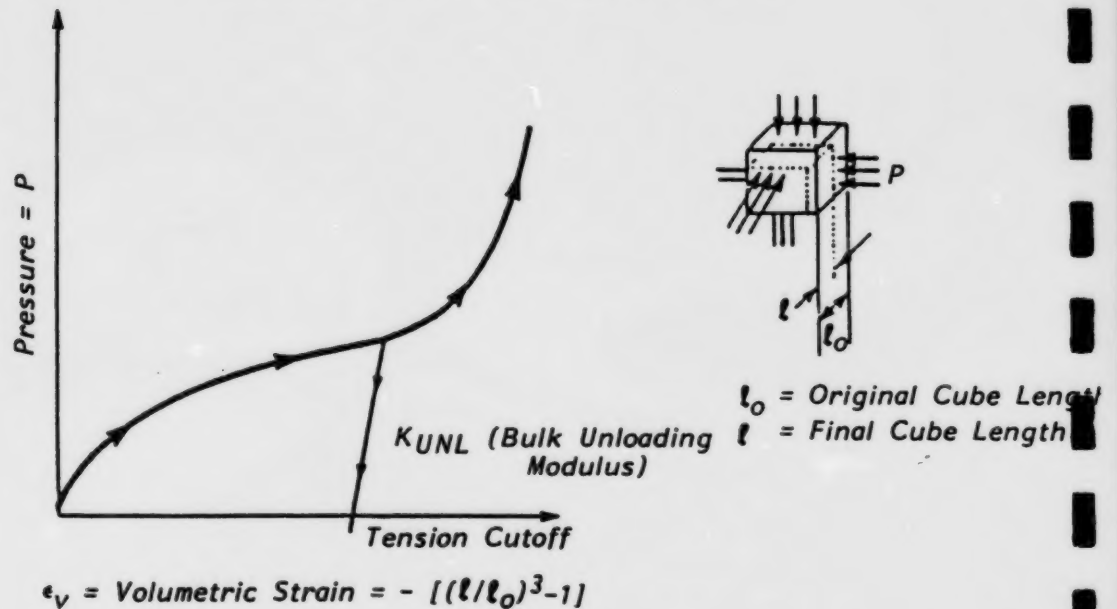


FIGURE 2.2.2
 PRESSURE/VOLUMETRIC STRAIN MEASURES
 CRUSHABLE FOAM MATERIAL MODEL

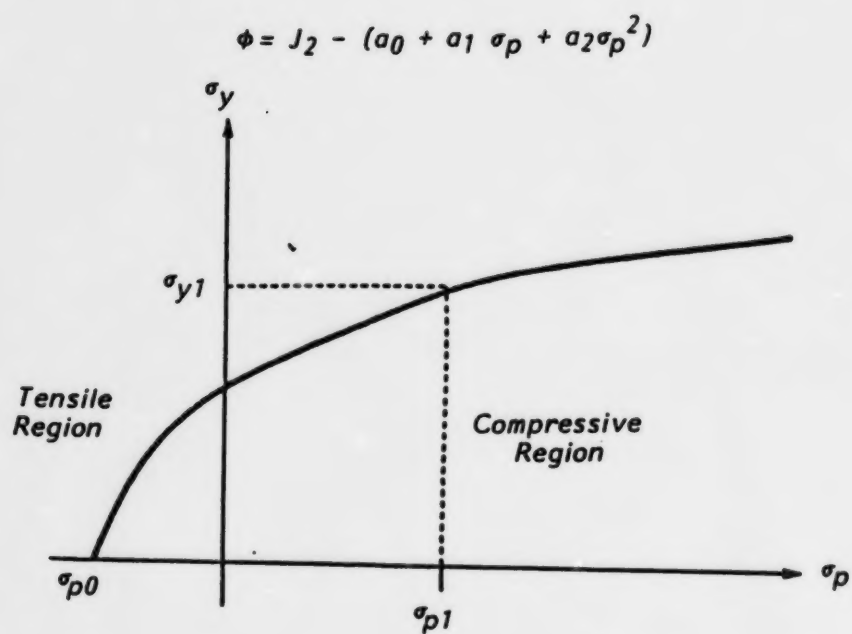


FIGURE 2.2.3
POLYURETHANE FOAM MODEL
PRESSURE VERSUS YIELD STRESS CURVE

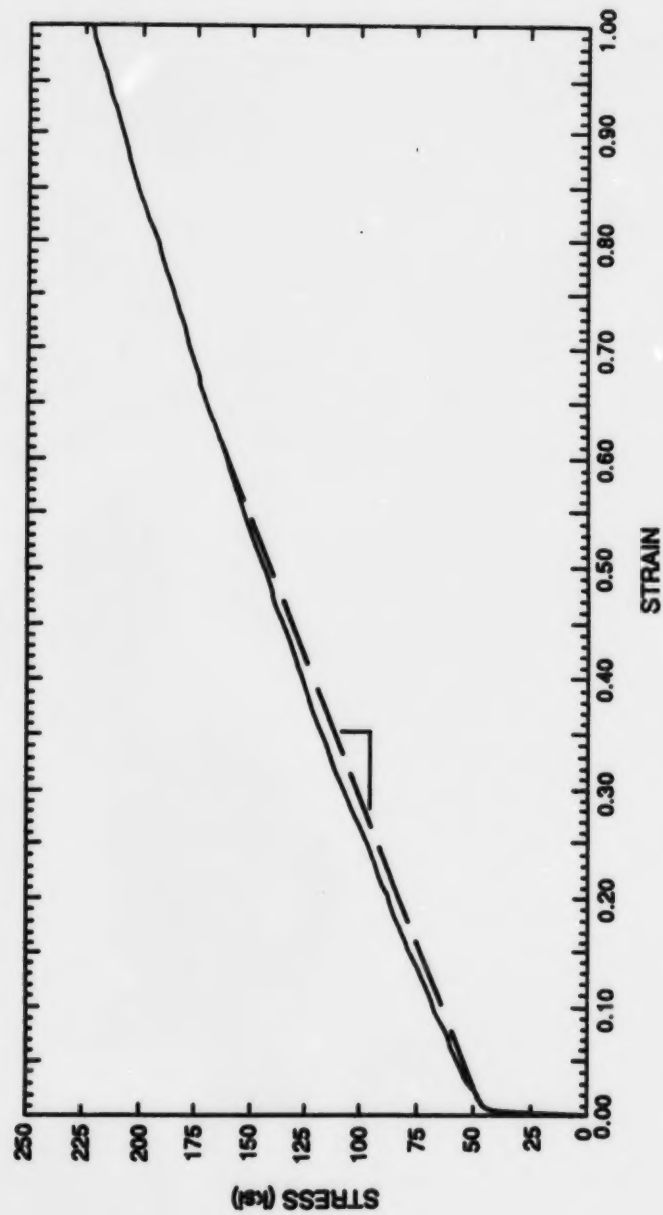


FIGURE 2.2.4
STRESS / STRAIN CURVE FOR 304L
STAINLESS STEEL MODEL

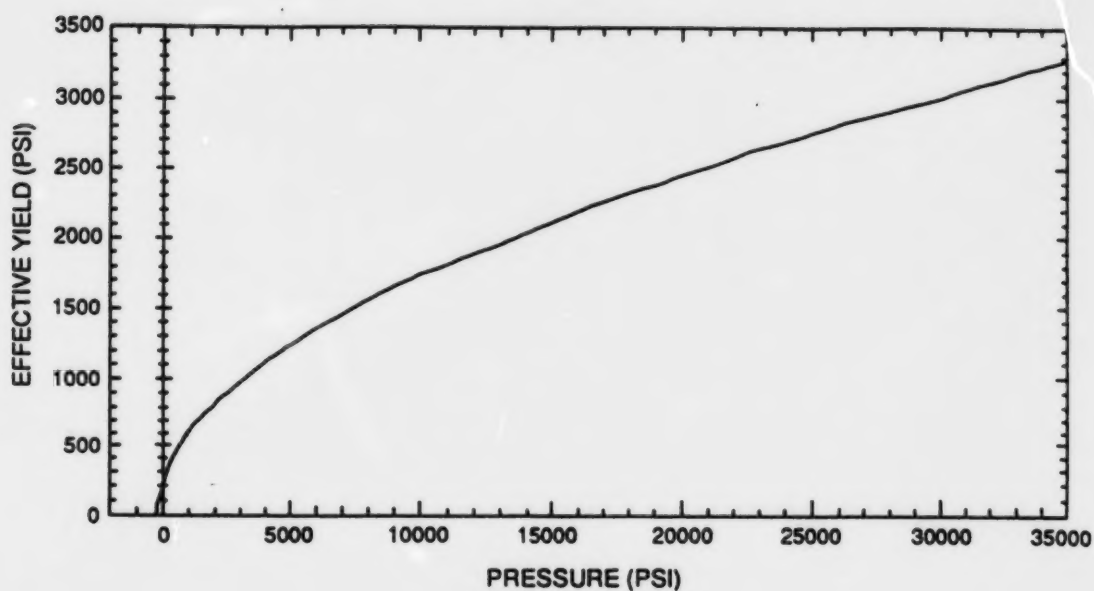


FIGURE 2.2.5

**YIELD VS PRESSURE CURVE FOR
POLYURETHANE FOAM MODEL**

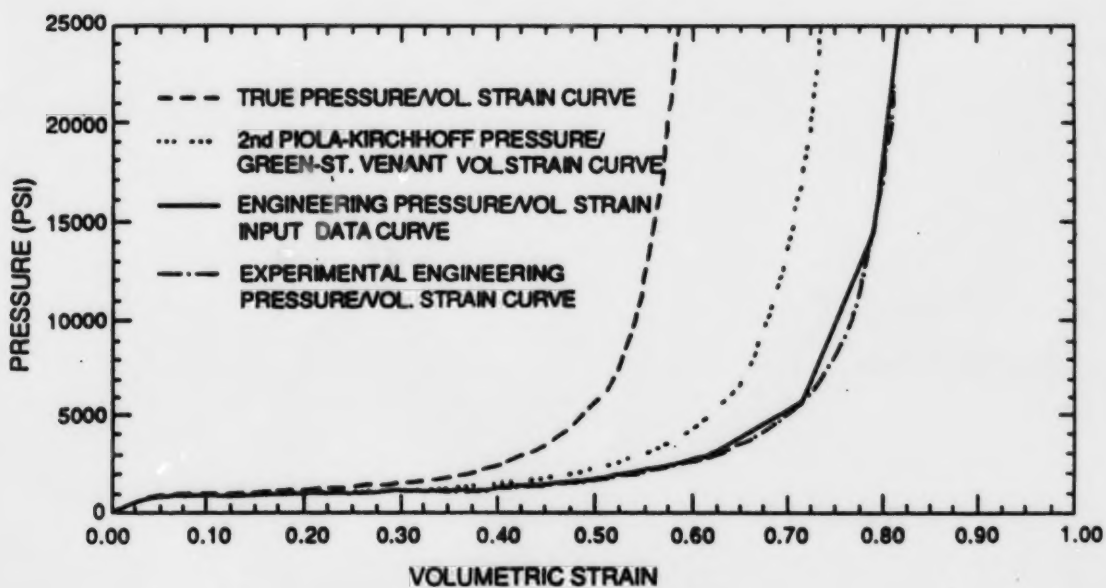


FIGURE 2.2.6

**PRESSURE / VOLUMETRIC STRAIN CURVE FOR
POLYURETHANE FOAM MODEL**

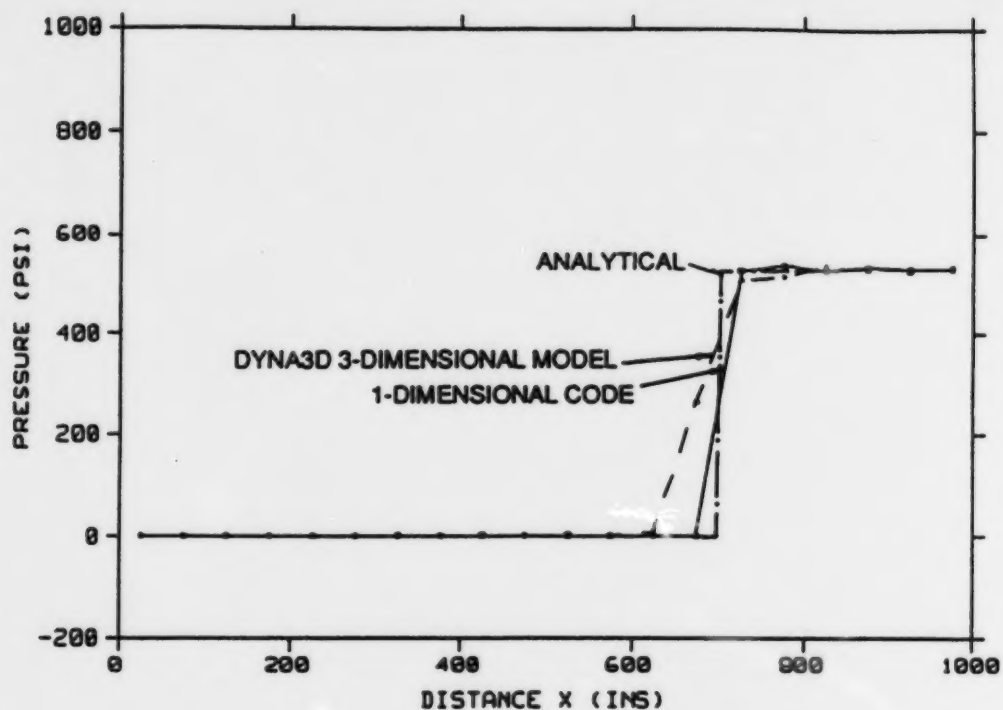


FIGURE 3.1

FLUID FORMULATION TEST -
PRESSURE DISTRIBUTION TIME = 0.005287 s

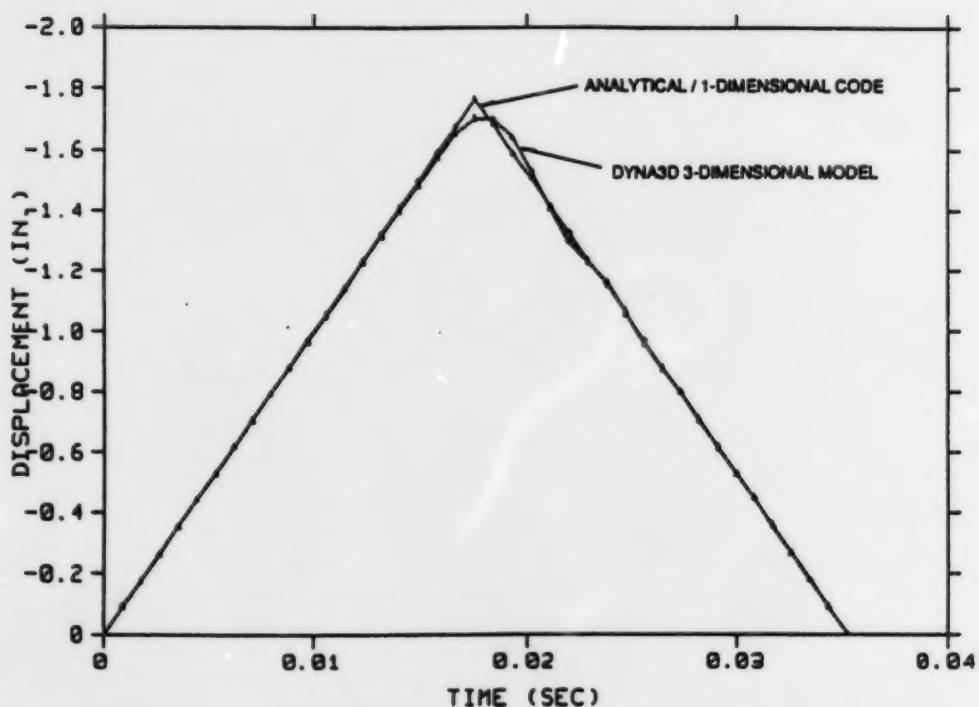


FIGURE 3.2

FLUID FORMULATION TEST -
DISPLACEMENT TIME HISTORY OF FREE SURFACE

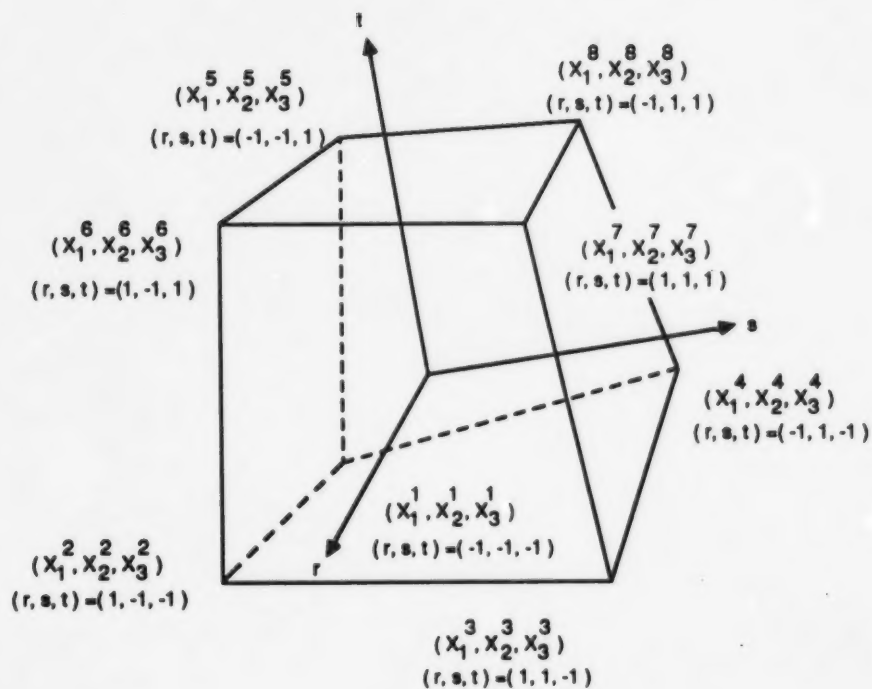


FIGURE 3.3
LINEAR ISOPARAMETRIC HEXAHEDRON ELEMENT

TH. 9M BOTTOM DOWN DROP F.S
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 0.0 PHI -90.0

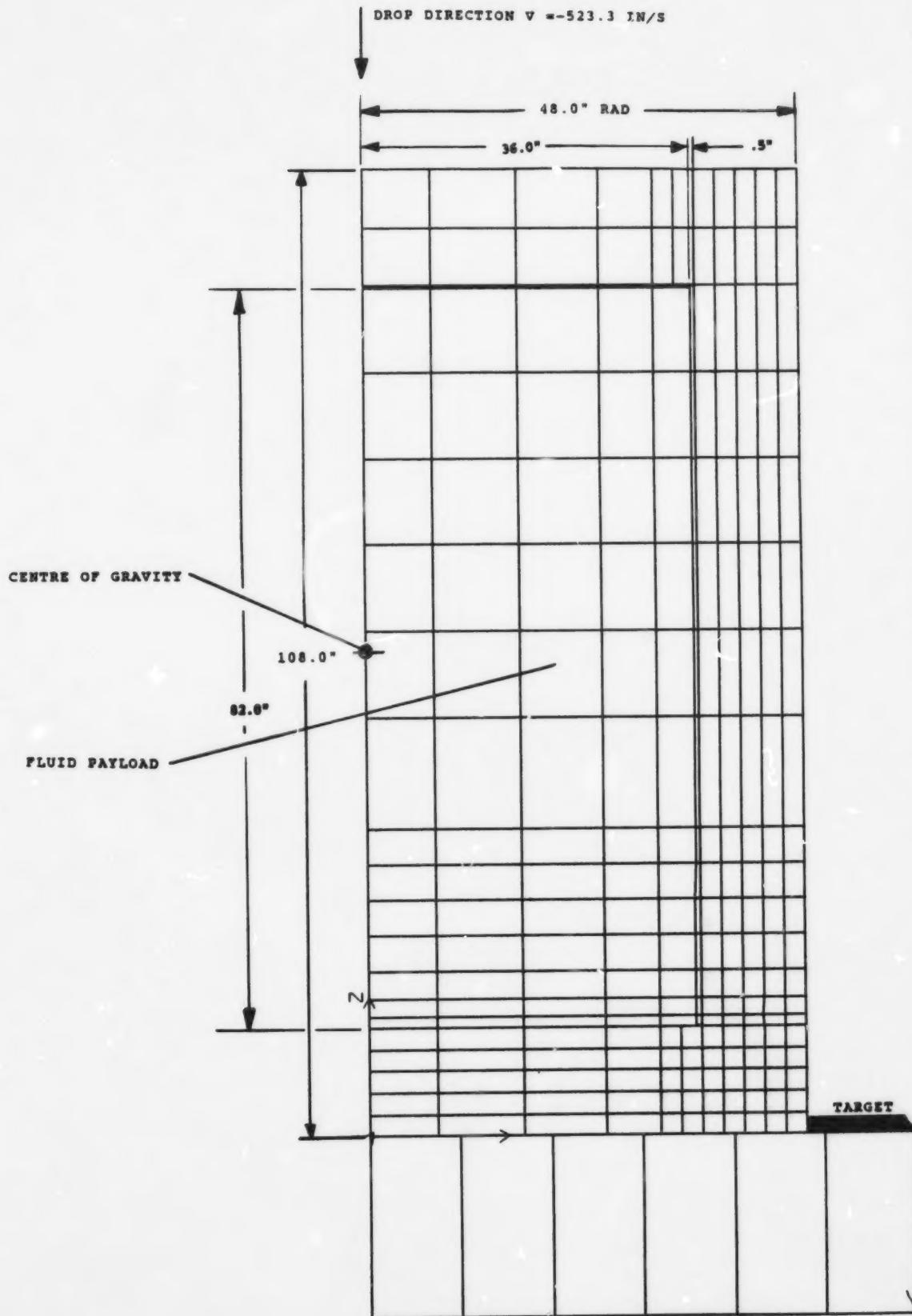


FIGURE 5.1.1
 TWTF FSX OVERALL MODELLING CONSIDERATION
 9M BOTTOM DOWN FLAT END DROP

TH: 9M BOTTOM DOWN DROP F.S
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 45.0 PHI -60.0

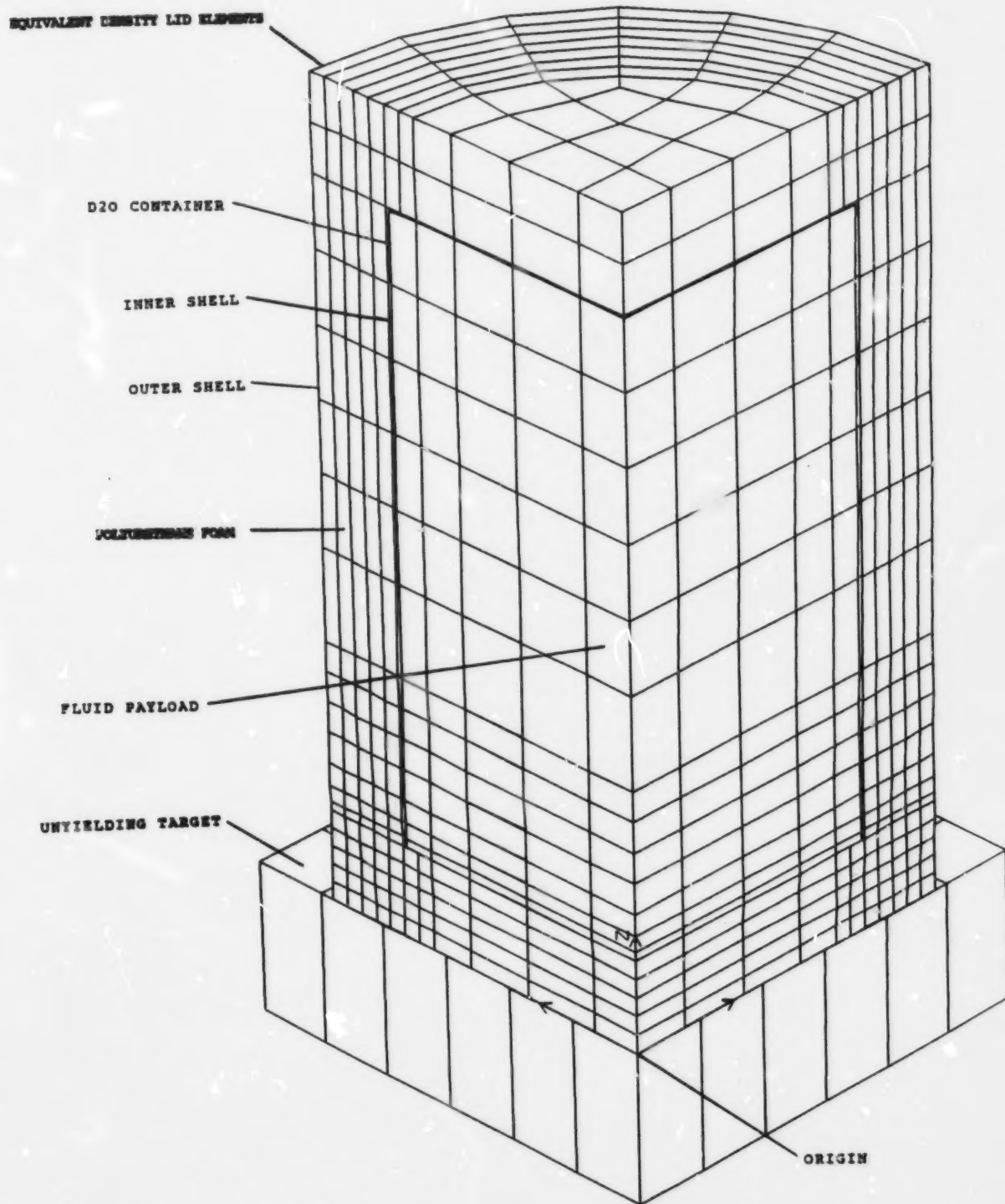


FIGURE 5.1.2
 OVERALL FULL SCALE TNTF PSI MODEL GEOMETRY

** THWTF LUID STRUCTURE INTERACTION MODEL NODES **
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 40.0 PHI -60.0

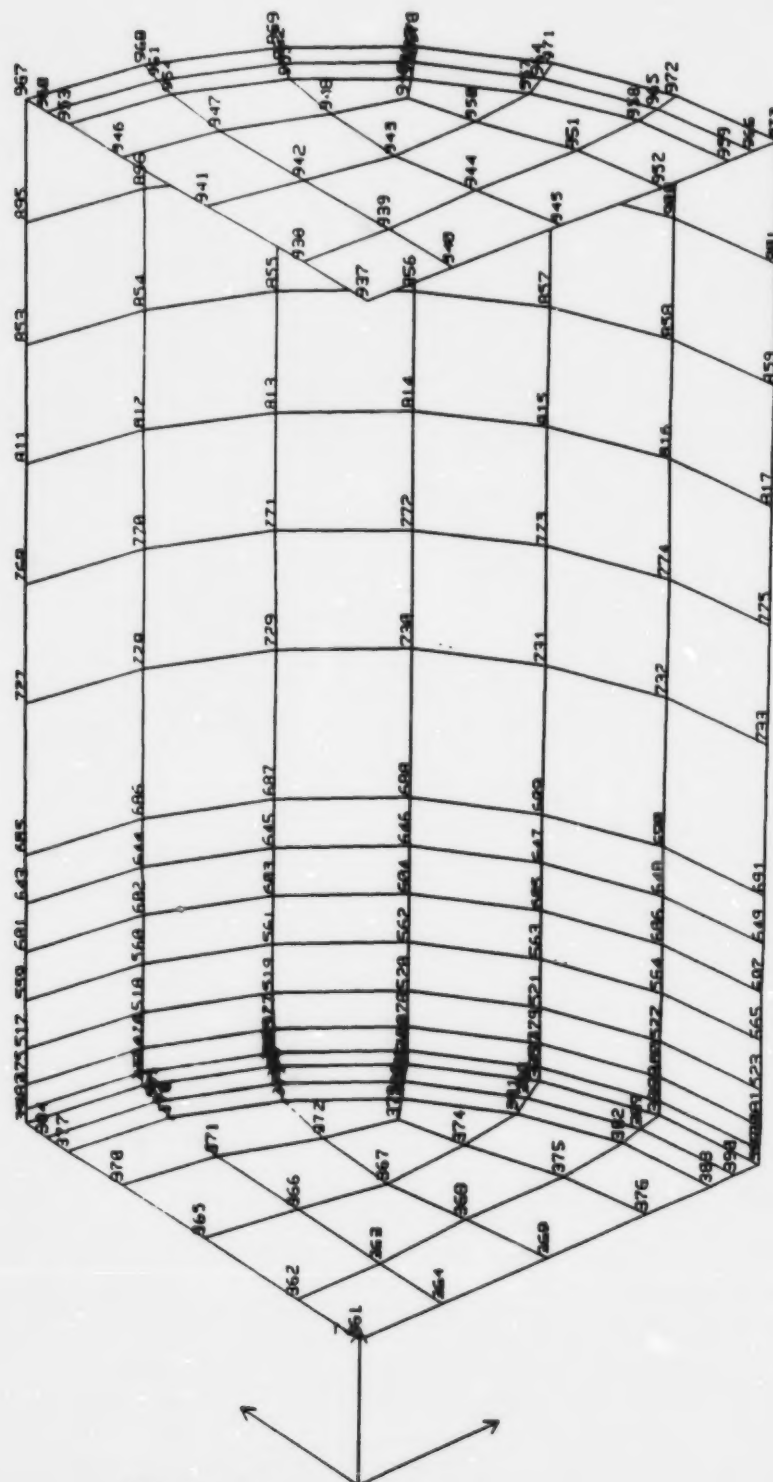


FIGURE 5.1.3
 OVERPACK BODY INNER LINER

** THWT. LUID STRUCTURE INTERACTION MODEL NODES **
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 40.0 PHI -50.0

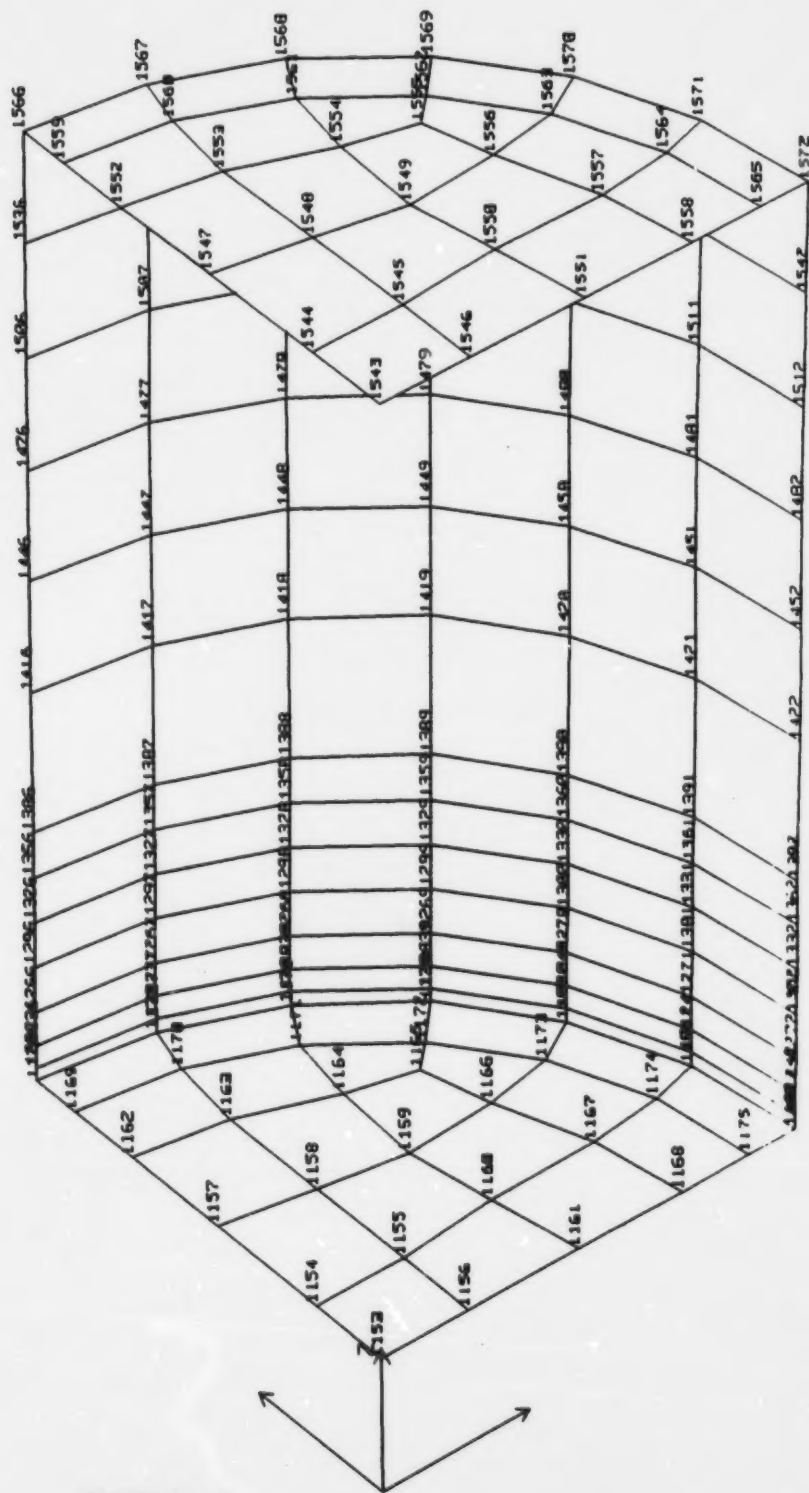


FIGURE 3.1.4
 D20 CONTAINER

** THWT, LUID STRUCTURE INTERACTION MODEL NODES **
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 40.0 PHI -60.0

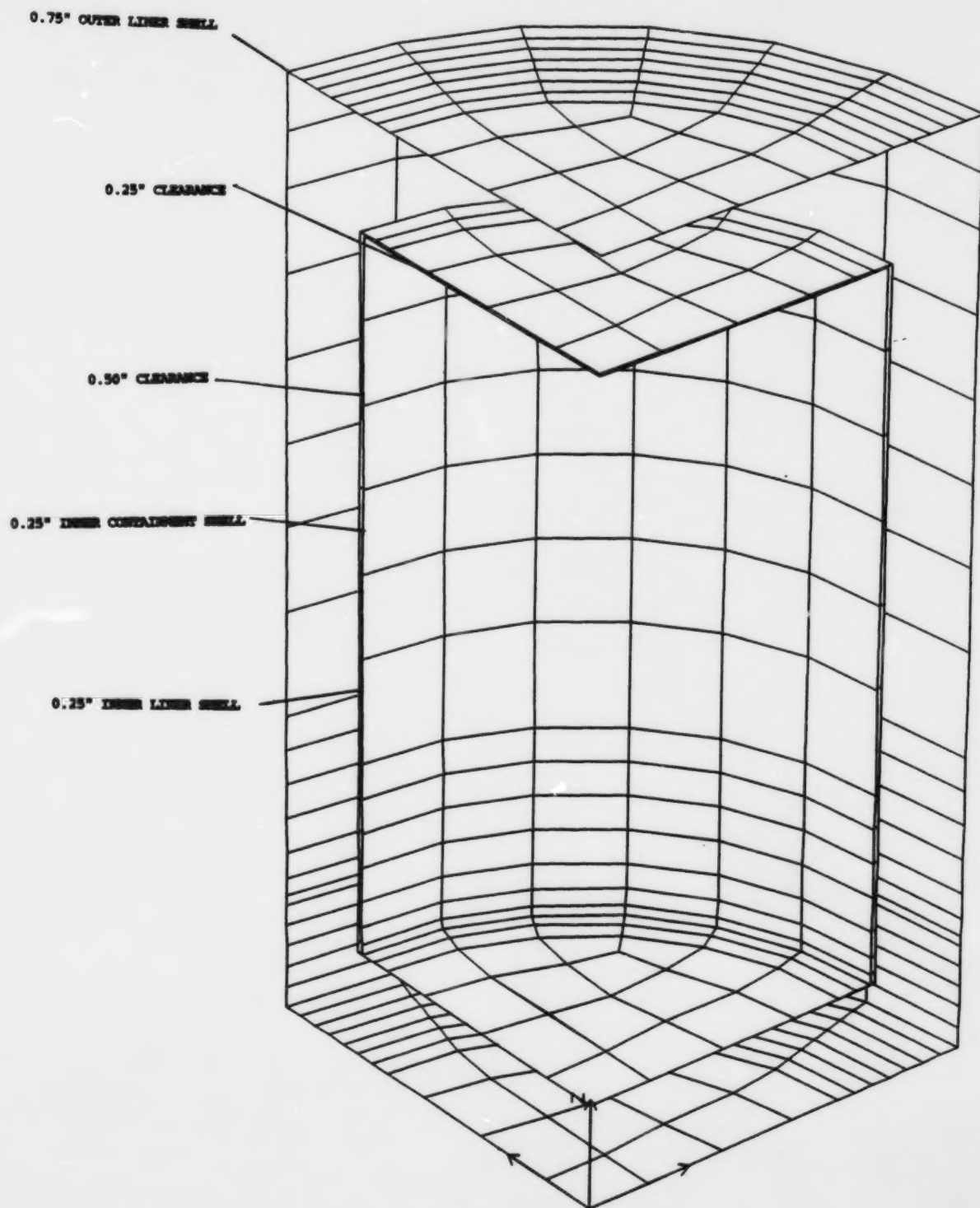


FIGURE 5.1.5
 THWT PSI SHELL ASSEMBLY

** THWTF - FLUID STRUCTURE INTERACTION MODEL NODES **
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 40.0 PHI -60.0

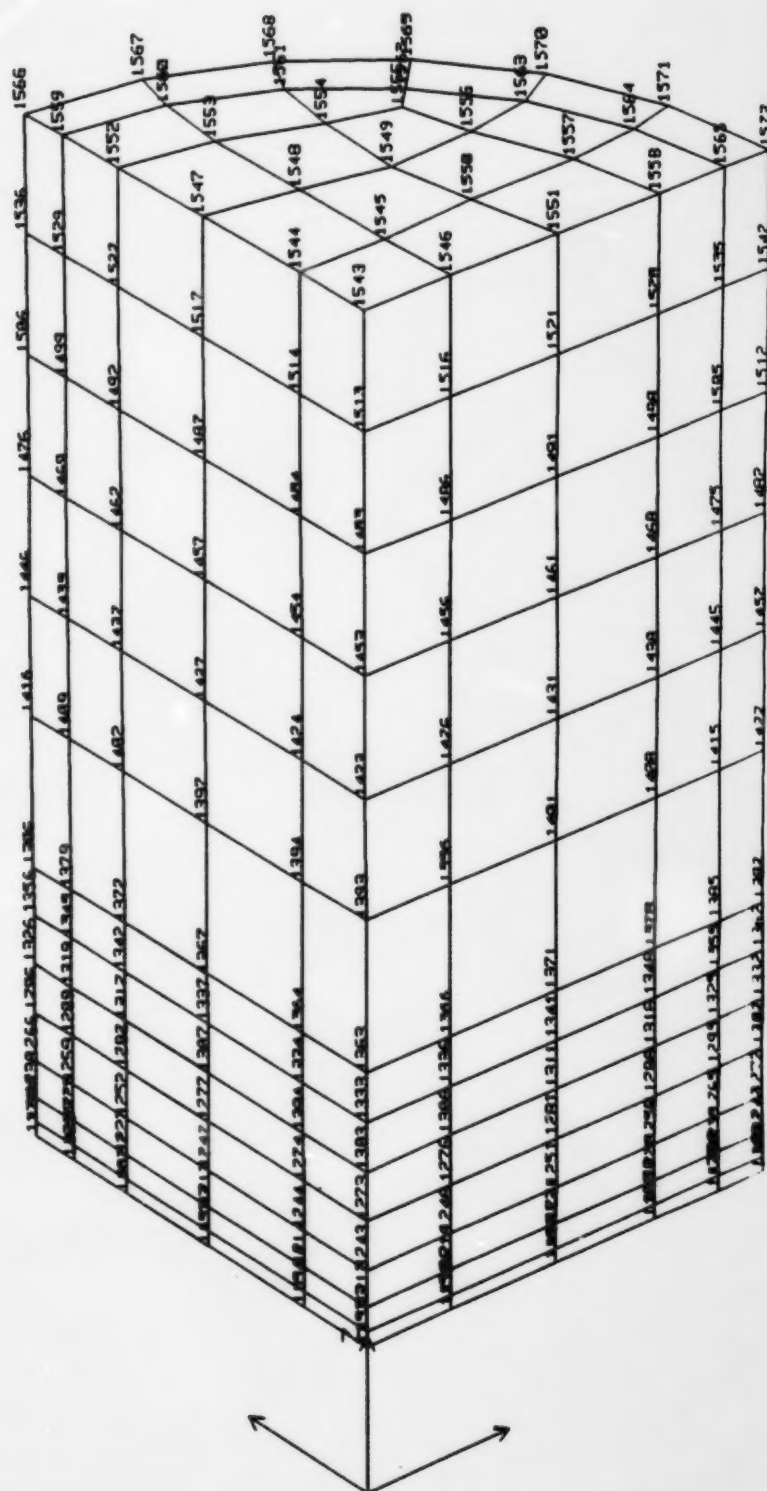


FIGURE 5.1.6
 FLUID ELEMENTS

** THMT: FLUID STRUCTURE INTERACTION MODEL NODES **
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 40.0 PHI -60.0

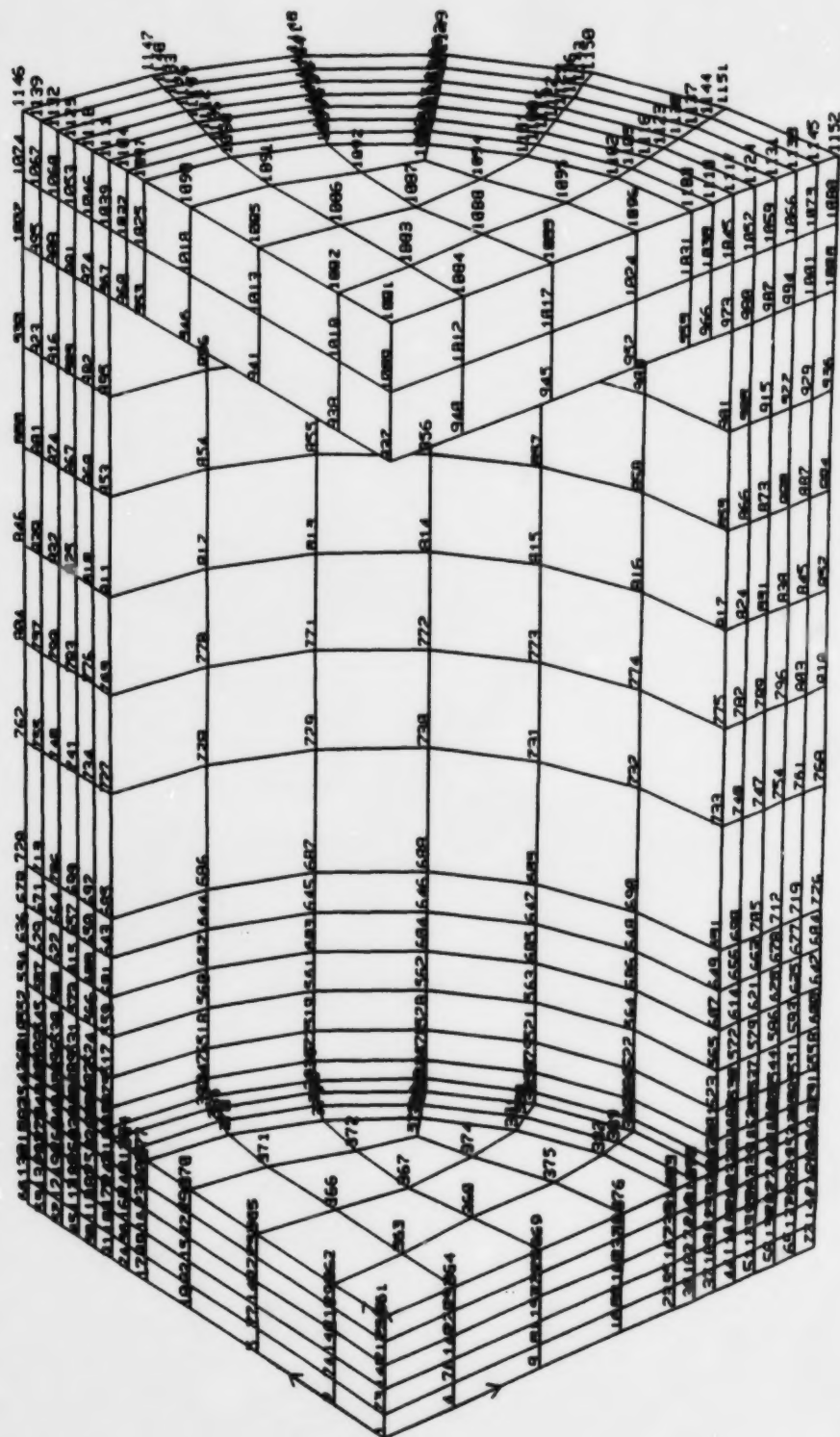


FIGURE 5.1.7
 FOAM OVERPACK BODY

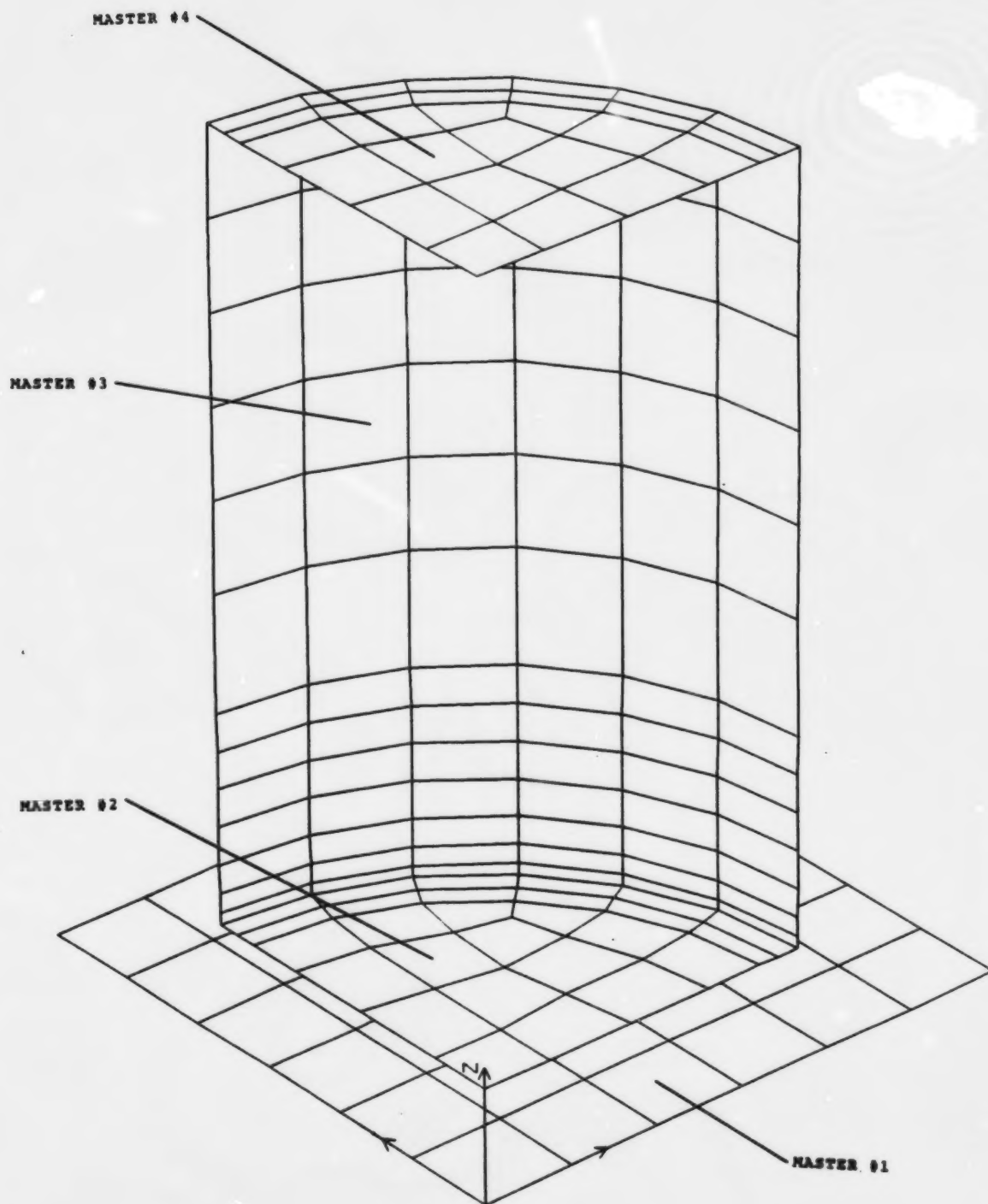


FIGURE 5.1.8
THWIP PSI MASTER SURFACES

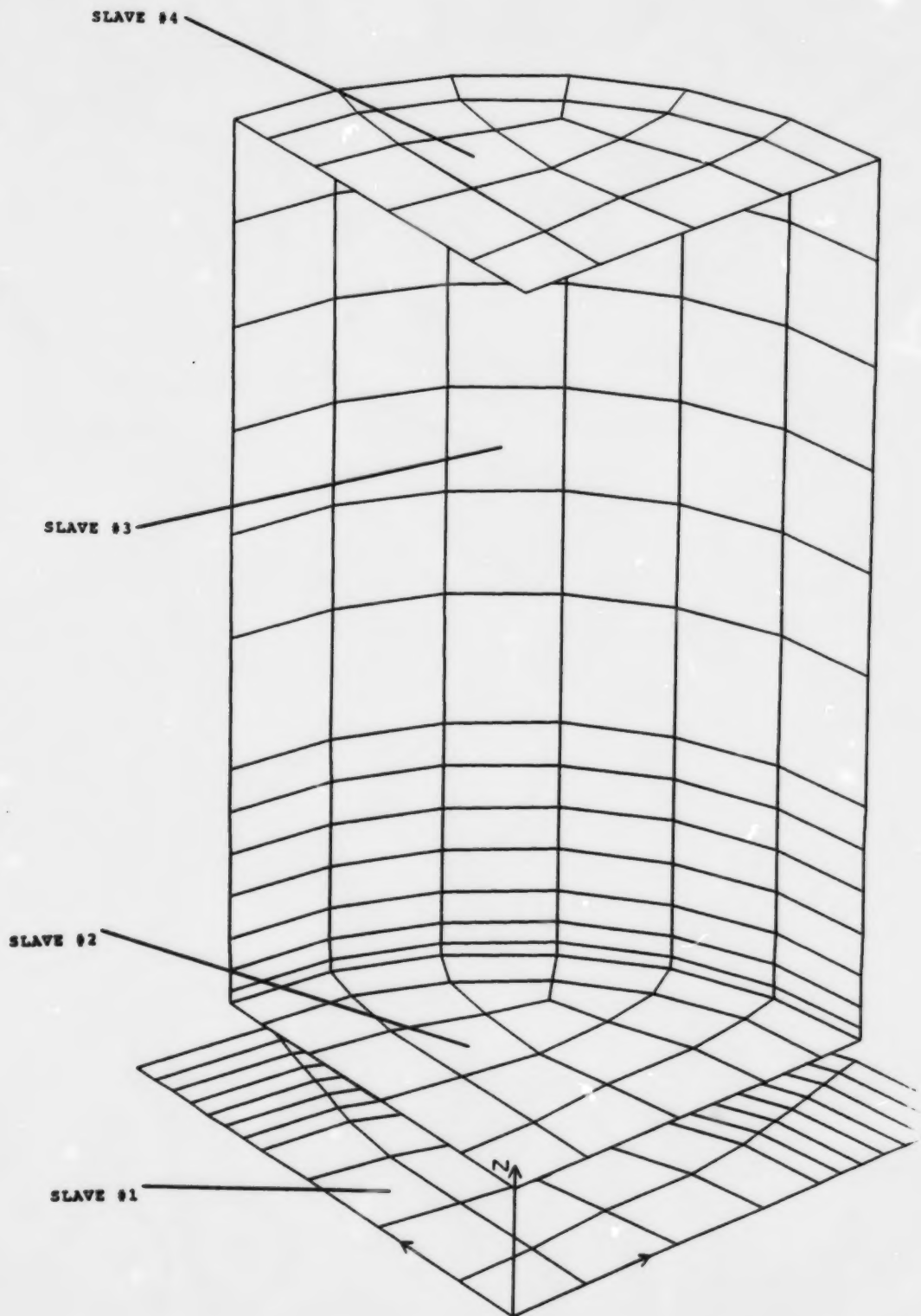


FIGURE 5.1.9

TWTF PSI SLAVE SURFACES

** D20 L SCALE FLUID TEST COORDINATES **
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 0.0 PHI 0.0

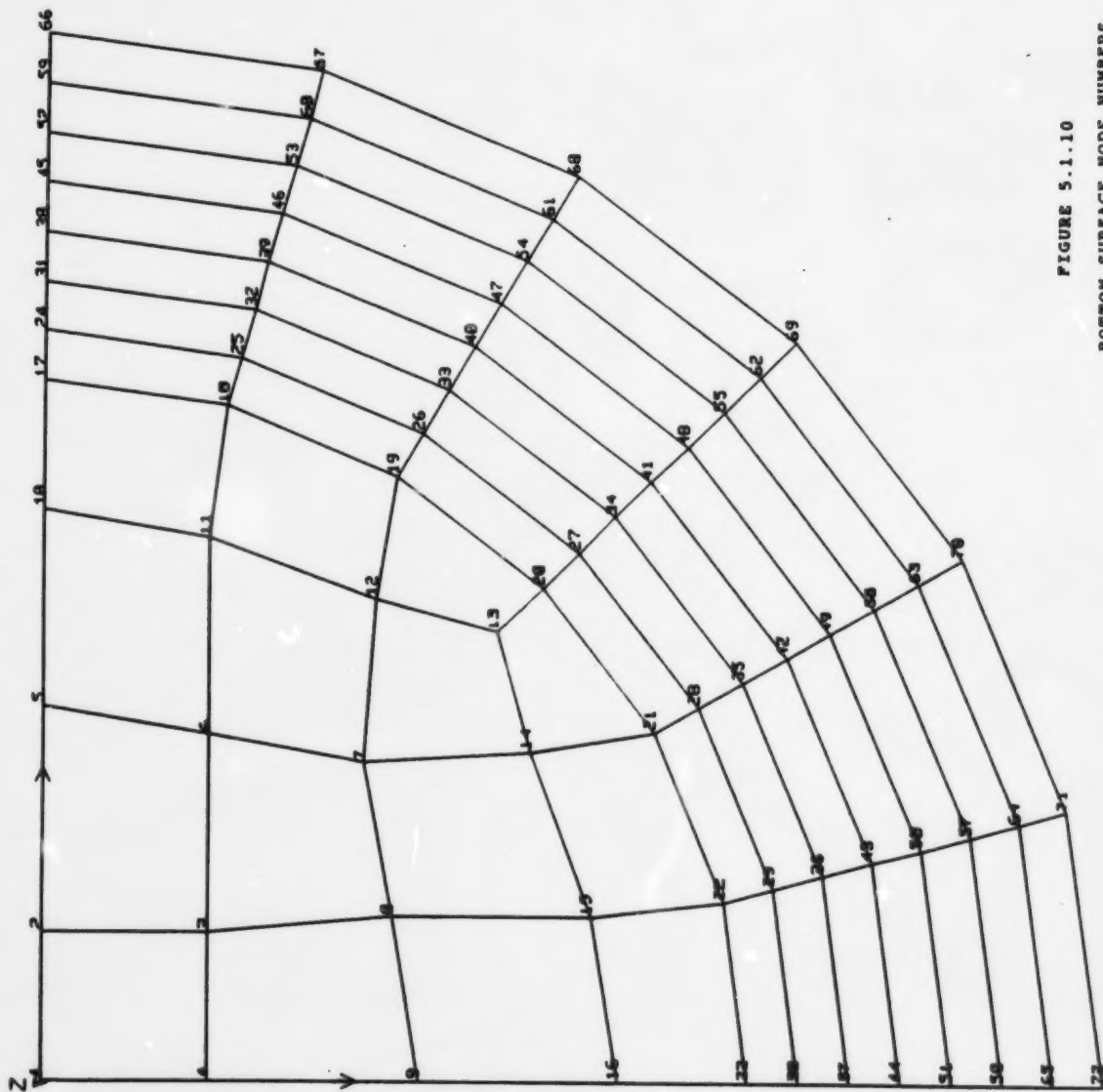


FIGURE 5.1.10
 BOTTOM SURFACE NODE NUMBERS

88 88
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 0.0 PHI 0.0

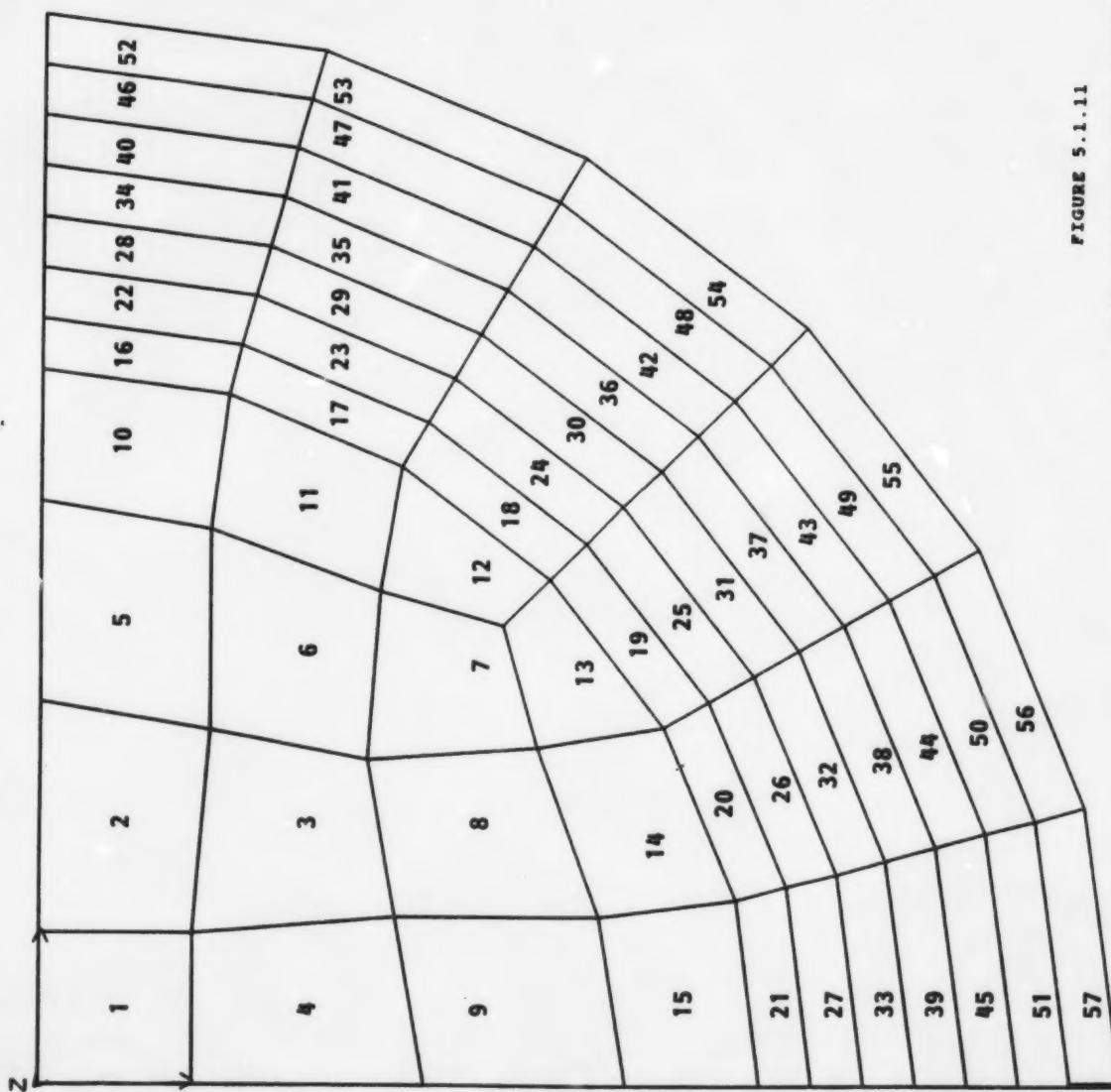


FIGURE 5.1.11
 BOTTOM SURFACE ELEMENT NUMBERS

TH. 9M BOTTOM DOWN DROP F.S
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 0.0 PHI 0.0

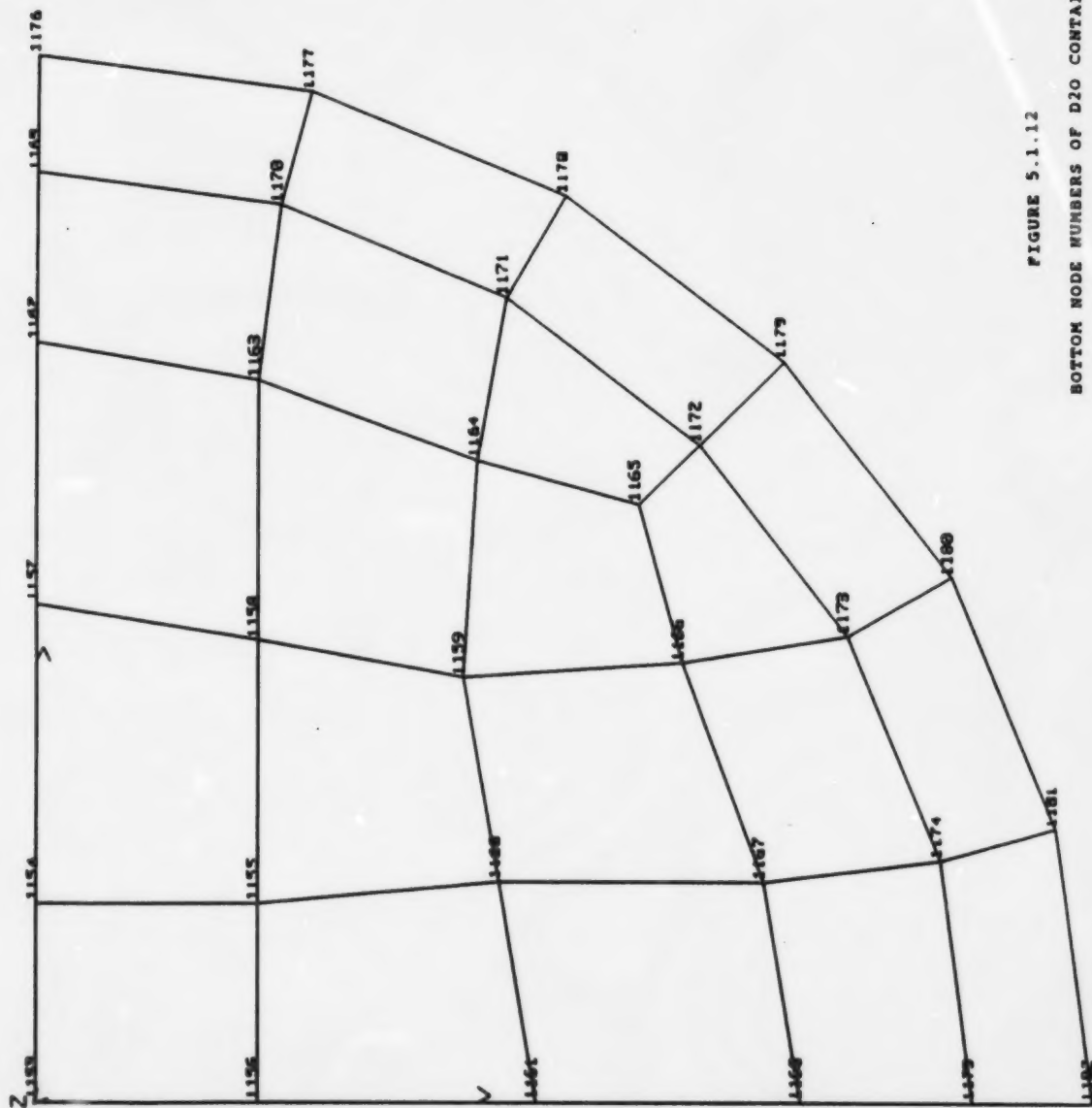


FIGURE 5.1.12
 BOTTOM NODE NUMBERS OF D20 CONTAINER

TH 5M BOTTOM DOWN DROP F.S
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 6.0 PHI 0.0

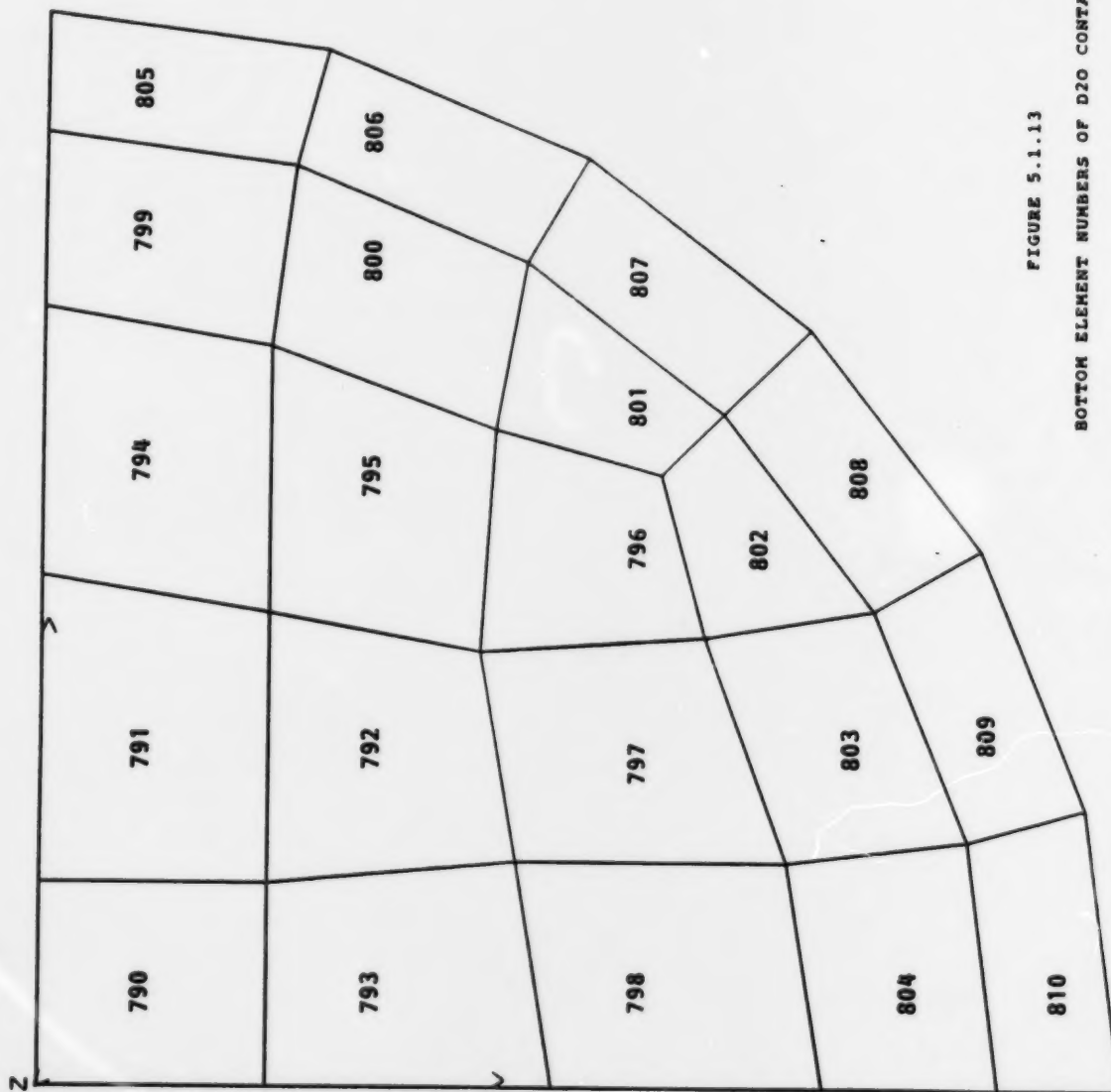


FIGURE 5.1.13

BOTTOM ELEMENT NUMBERS OF D20 CONTAINER

** D20 .L SCALE FLUID TEST COORDINATES **
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 45.0 PHI -45.0

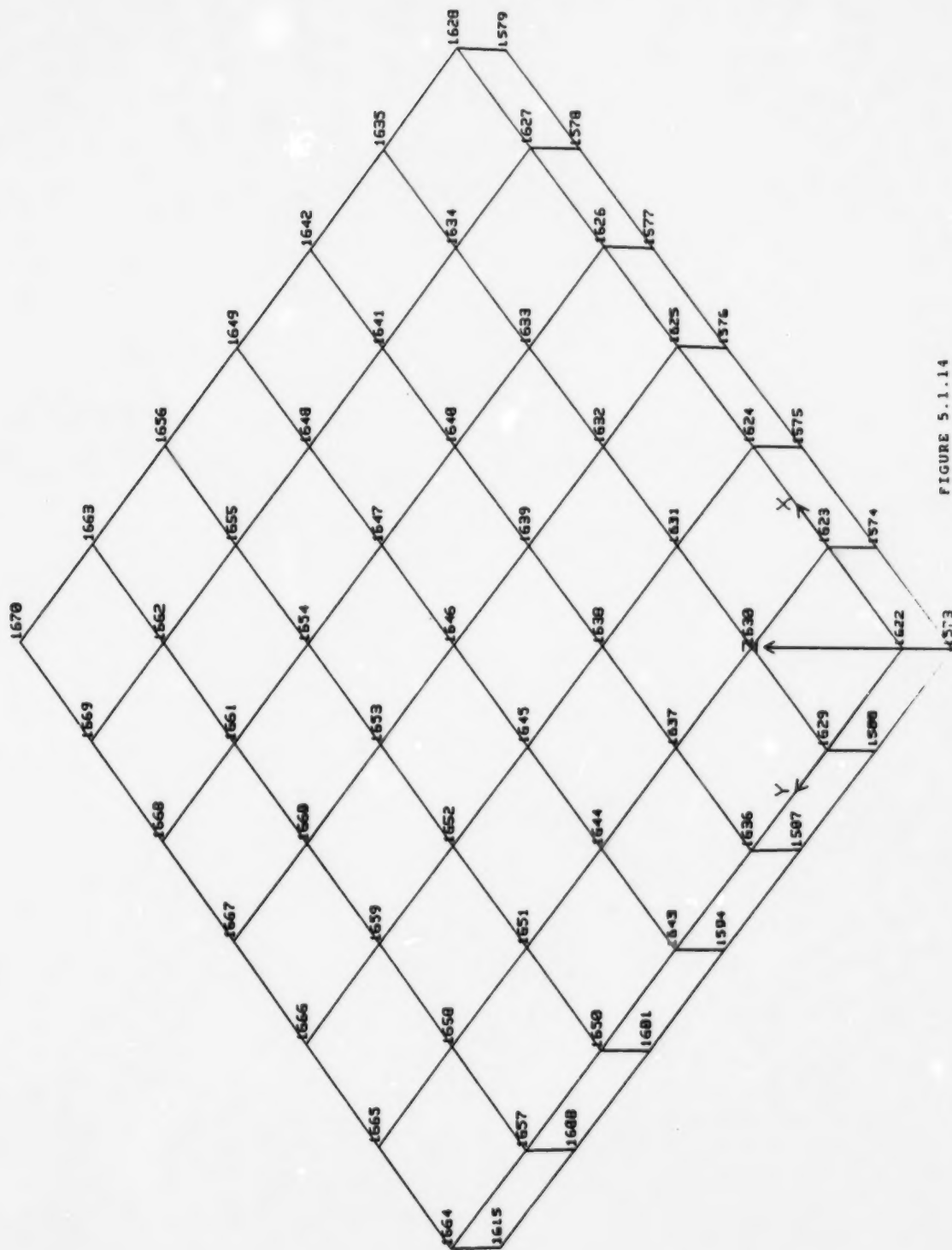


FIGURE 5.1.14

TARGET NODE NUMBERS

THIN, 9M BOTTOM DOWN DROP F.S
 UNDEFORMED GEOMETRY
 VIEW ANGLE: THETA 45.0 PHI -45.0

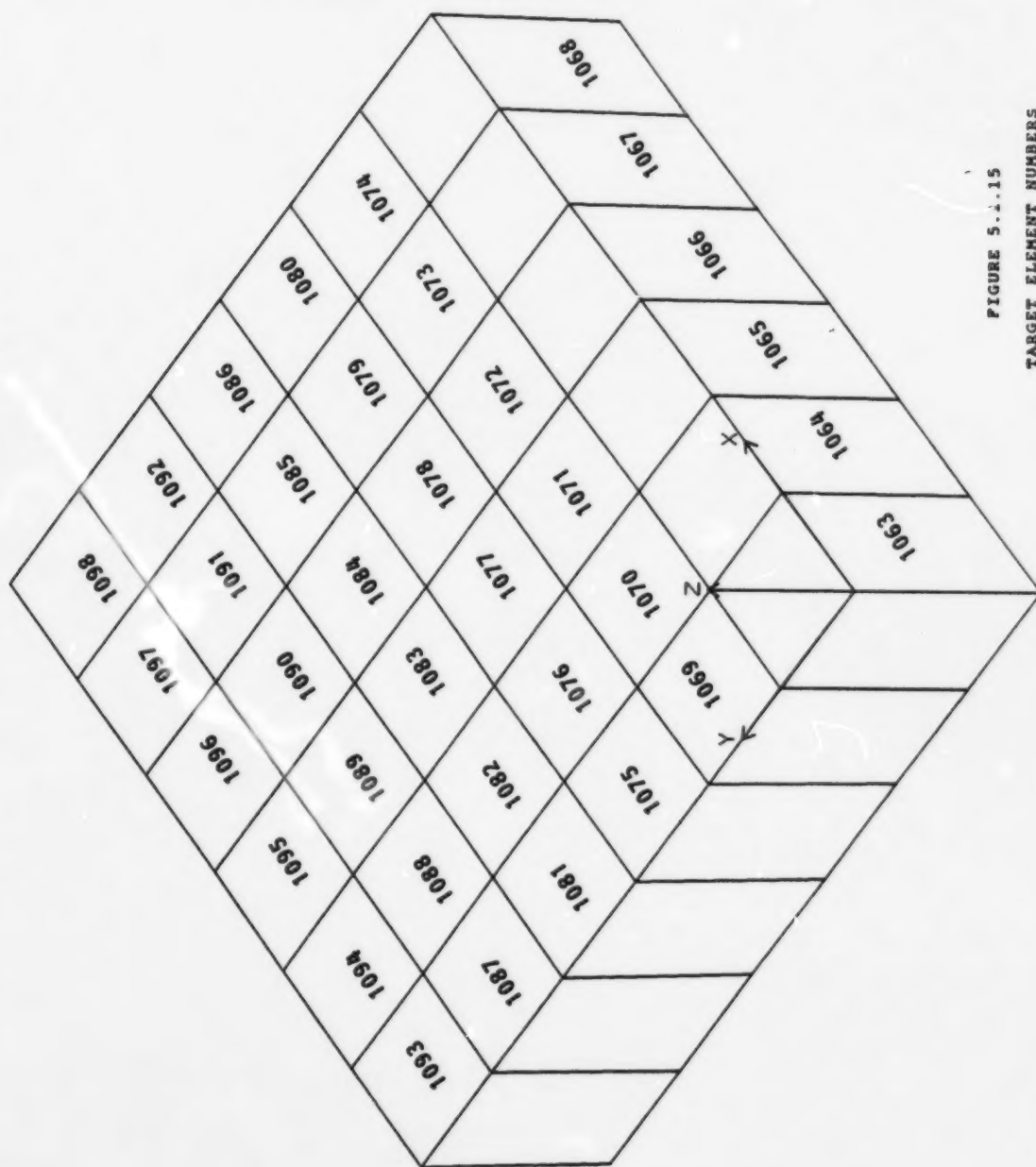


FIGURE S.1.15
 TARGET ELEMENT NUMBERS

TH, 9M BOTTOM DOWN DROP F.S
TIME= 0.001996661
VIEW ANGLE: THETA 0.0 PHI -90.0

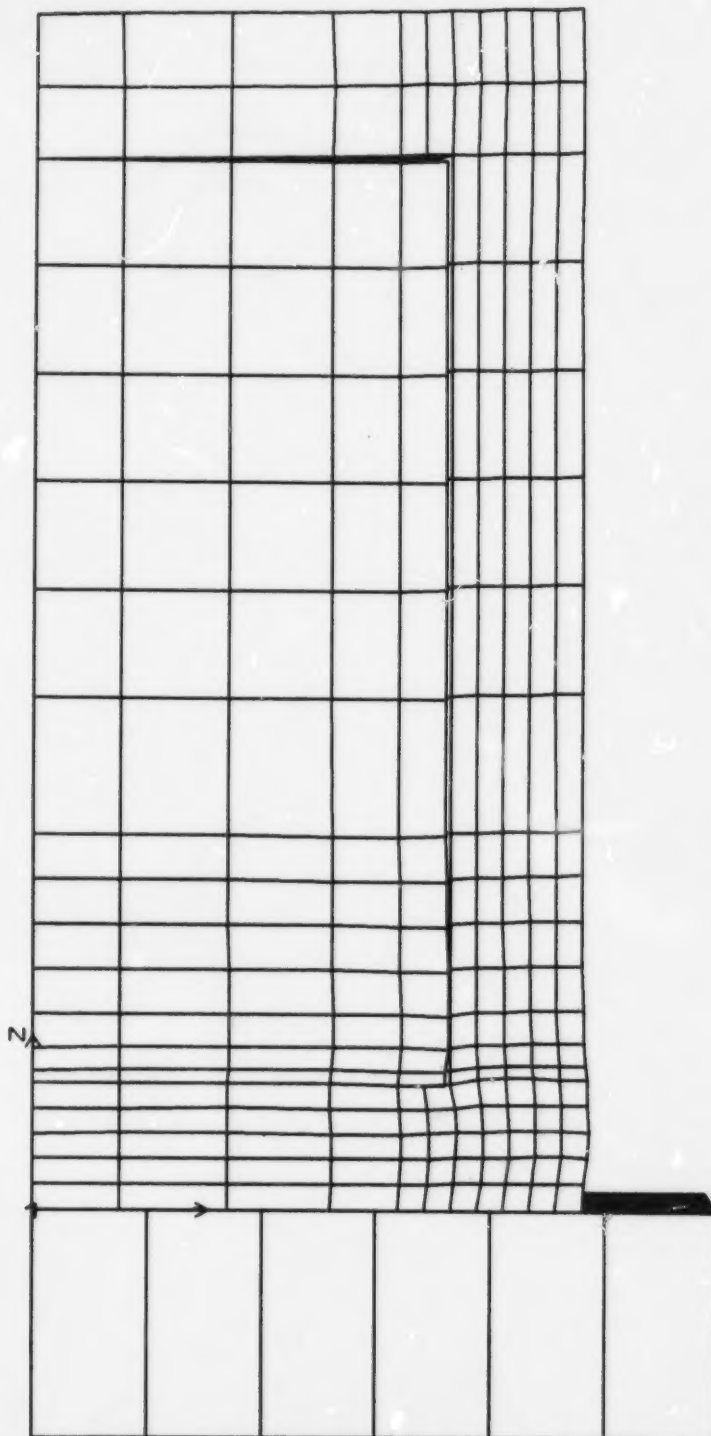


FIGURE 5.2.1

DEFORMED VIEW AT CUT PLANE (Y=0.0, 0)
TIME= 0.001996661 SEC

T_h ' 9M BOTTOM DOWN DROP F.S
 TIME= 0.002997127
 VIEW ANGLE: THETA 0.0 PHI -90.0

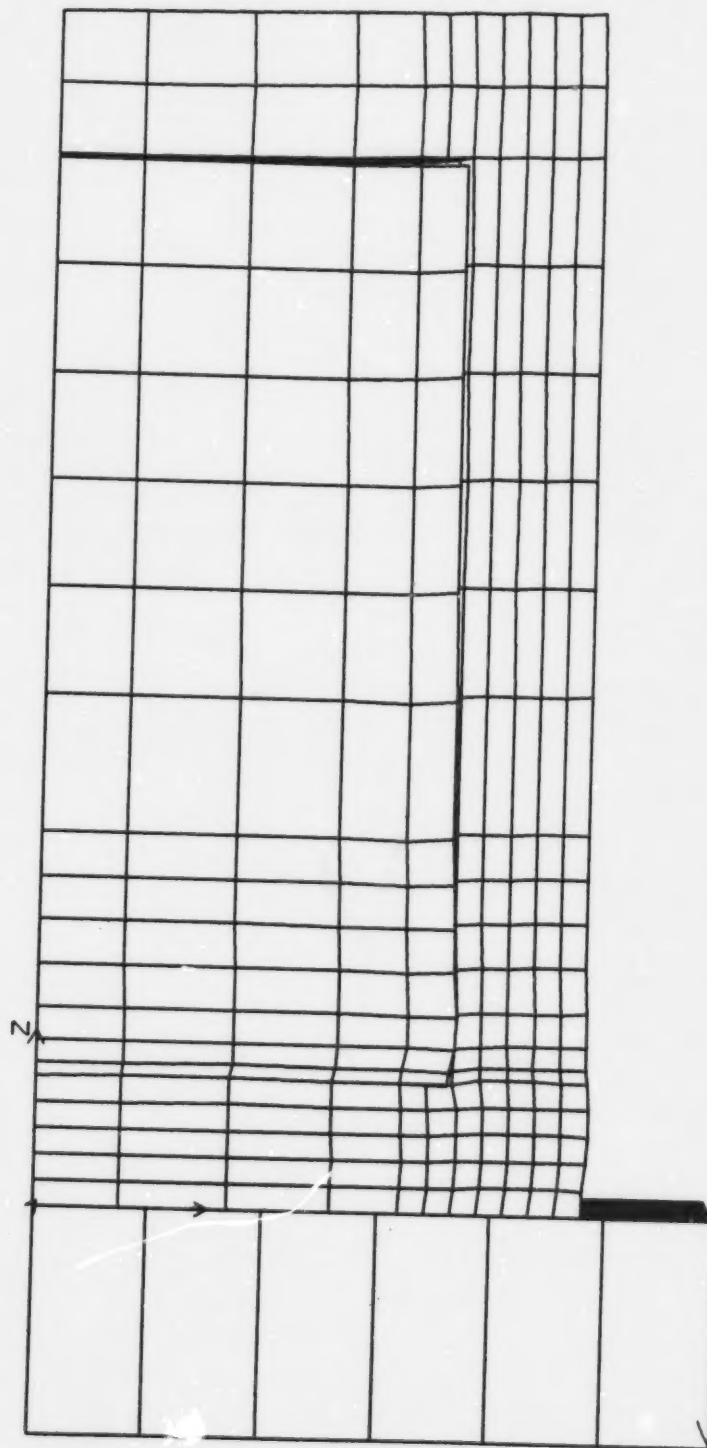


FIGURE 5.2.2

DEFORMED VIEW AT CUT PLANE (Y=0.,0.)
 TIME=0.002997127 SEC

TH. 9M BOTTOM DOWN DROP F.S
 TIME= 0.004998096
 VIEW ANGLE: THETA 0.0 PHI -90.0

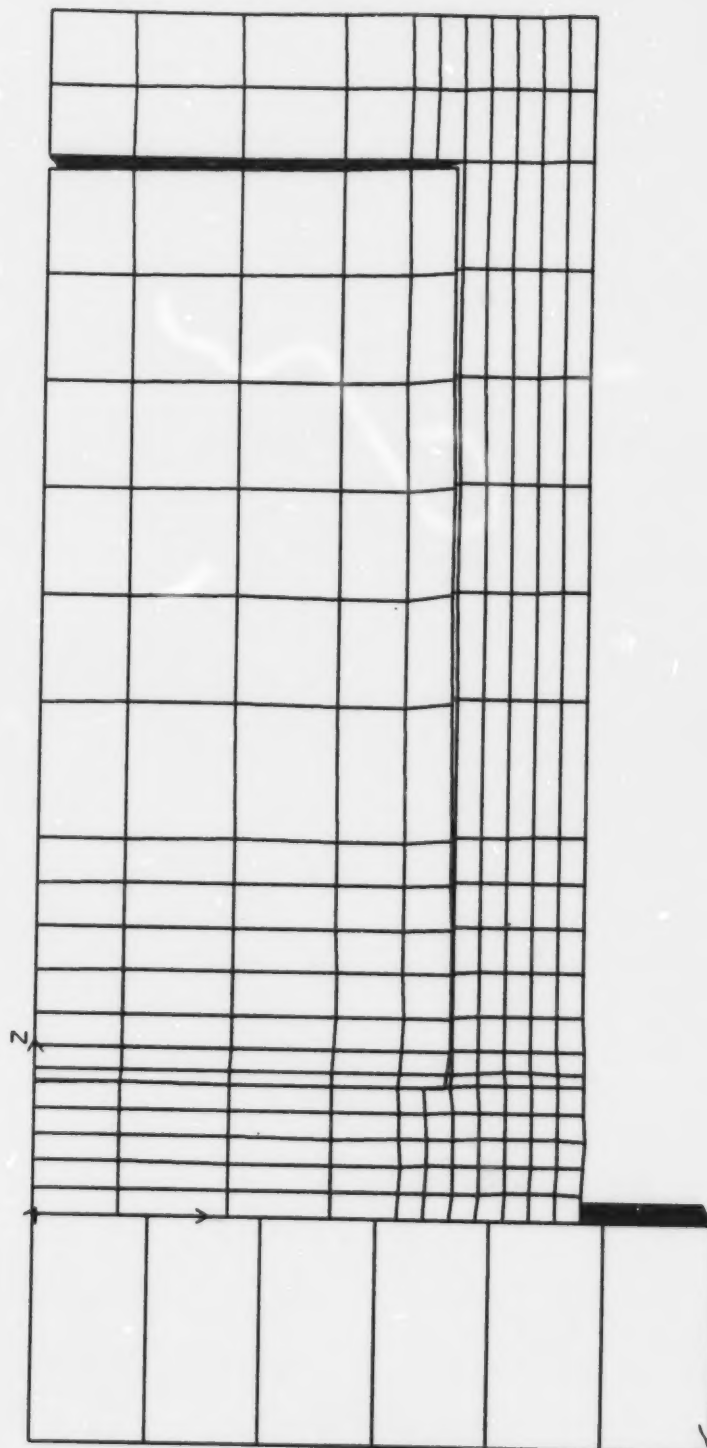


FIGURE 5.2.3

DEFORMED VIEW AT CUT PLANE (Y=0.,0.)
 TIME=0.004998096 SEC

THWTP FSI 9M BOTTOM DOWN FLAT DROP - KE

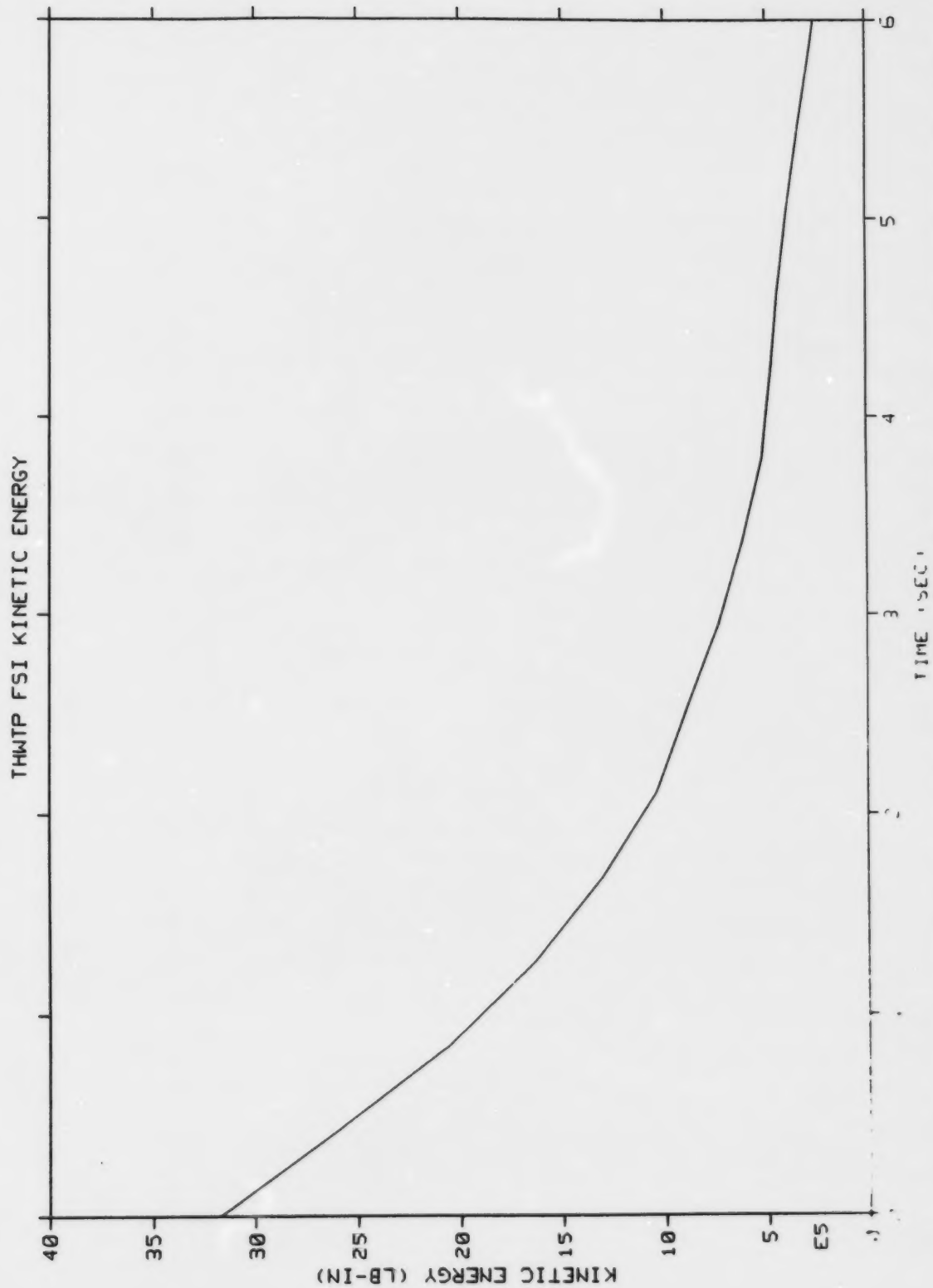


FIGURE 2.4

THWTP 9M BOTTOM DOWN F.S. IMPACT FORCES

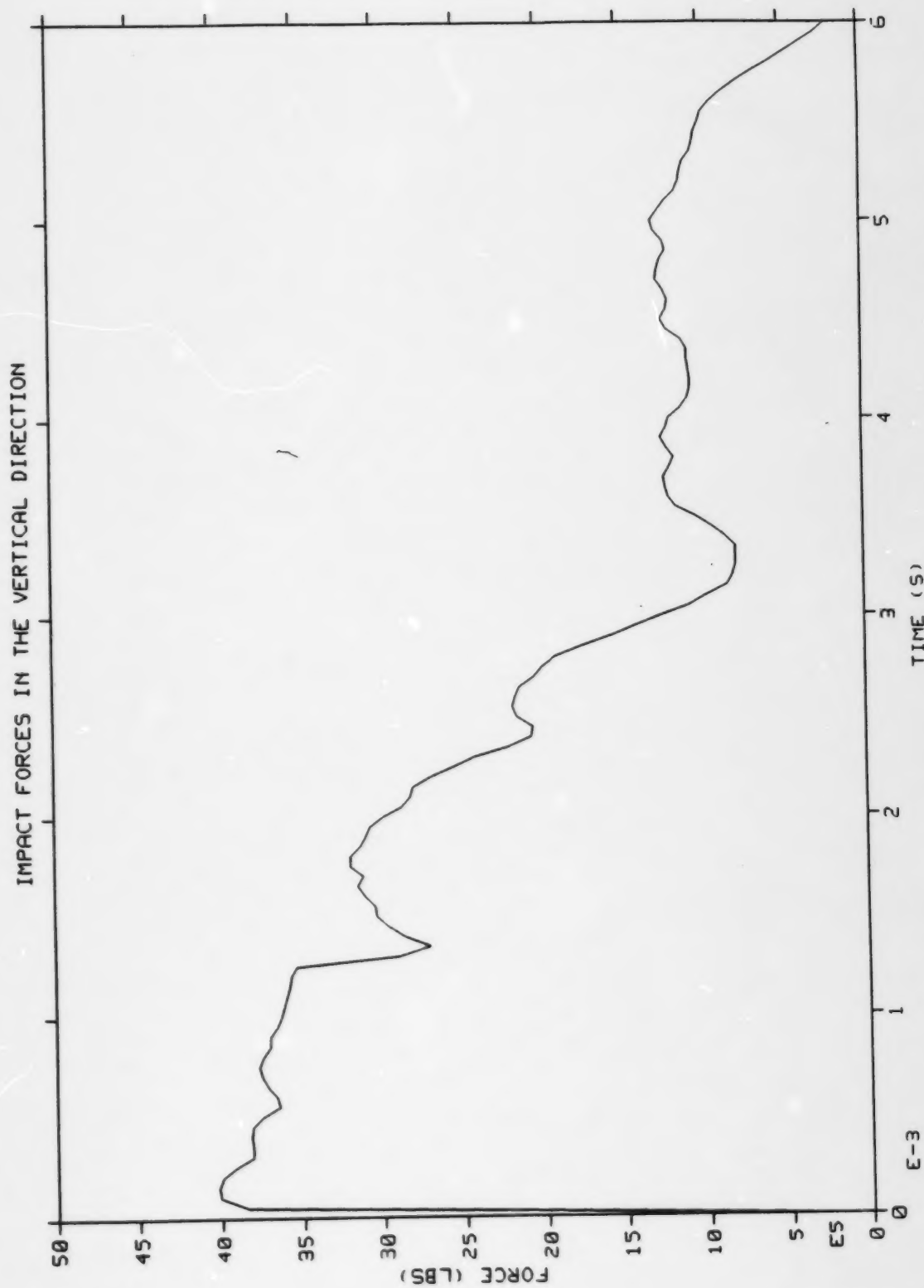
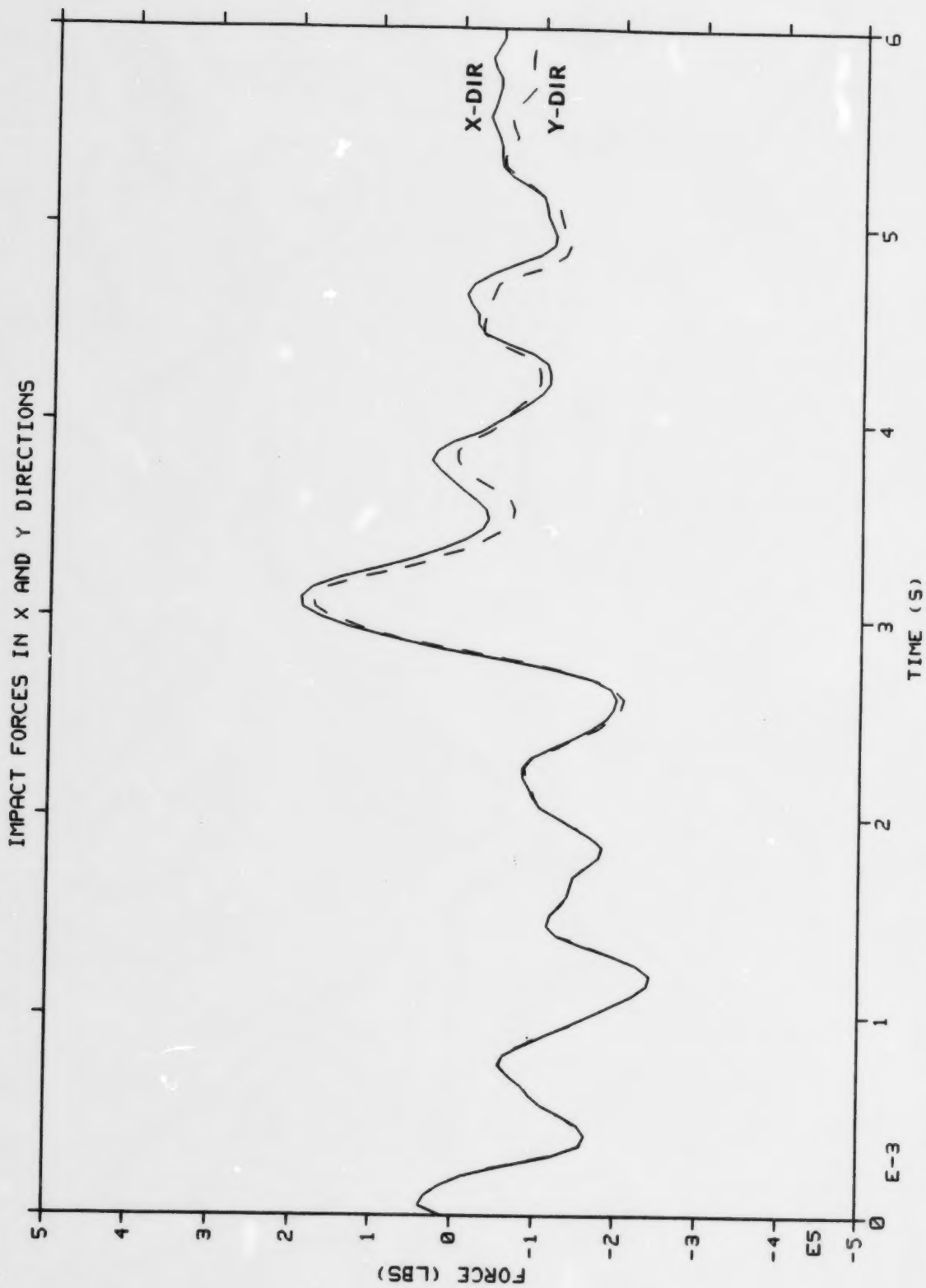


FIGURE 5.2.5

THUTP 9M BOTTOM DOWN F.S. IMPACT FORCES



THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS

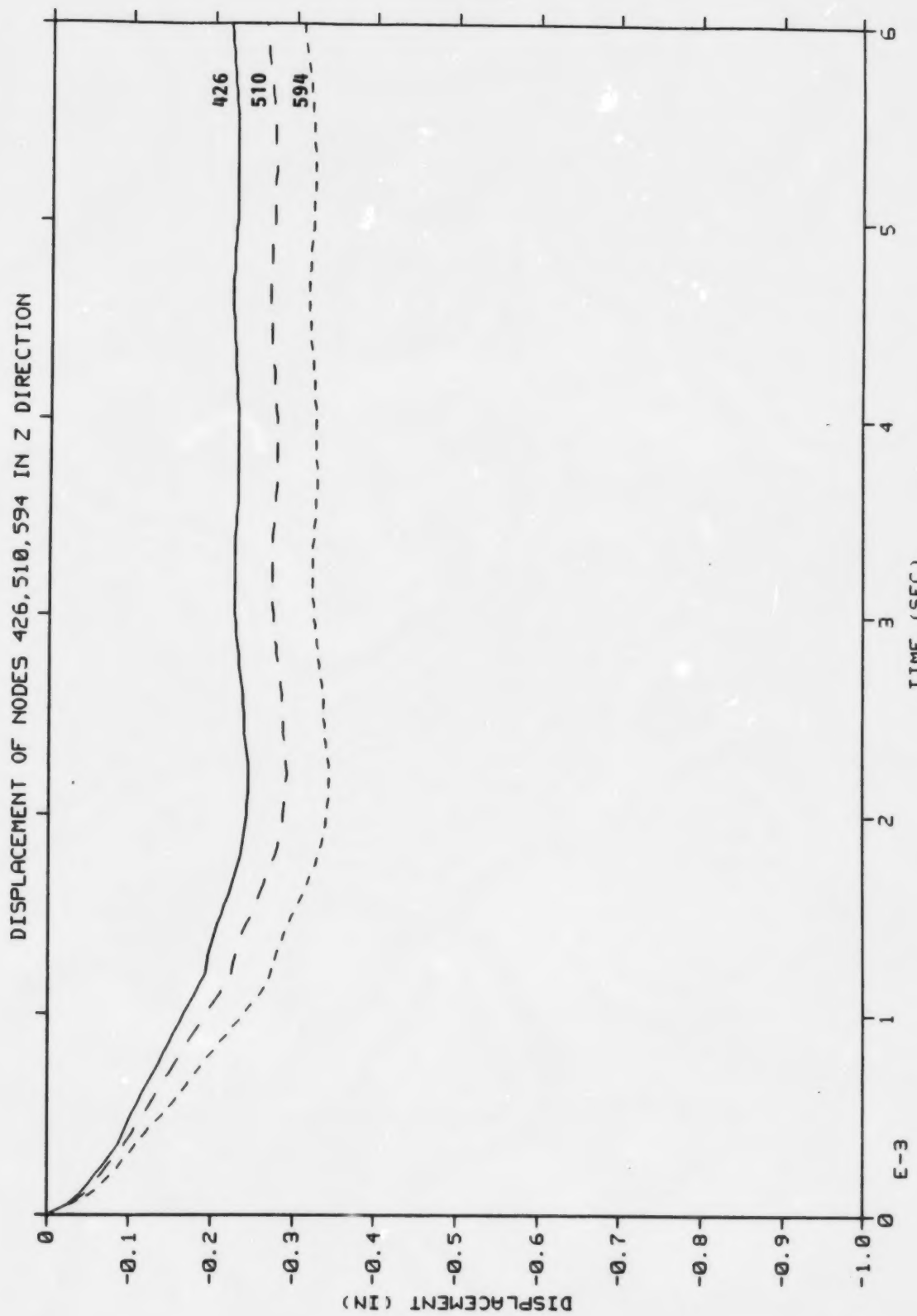
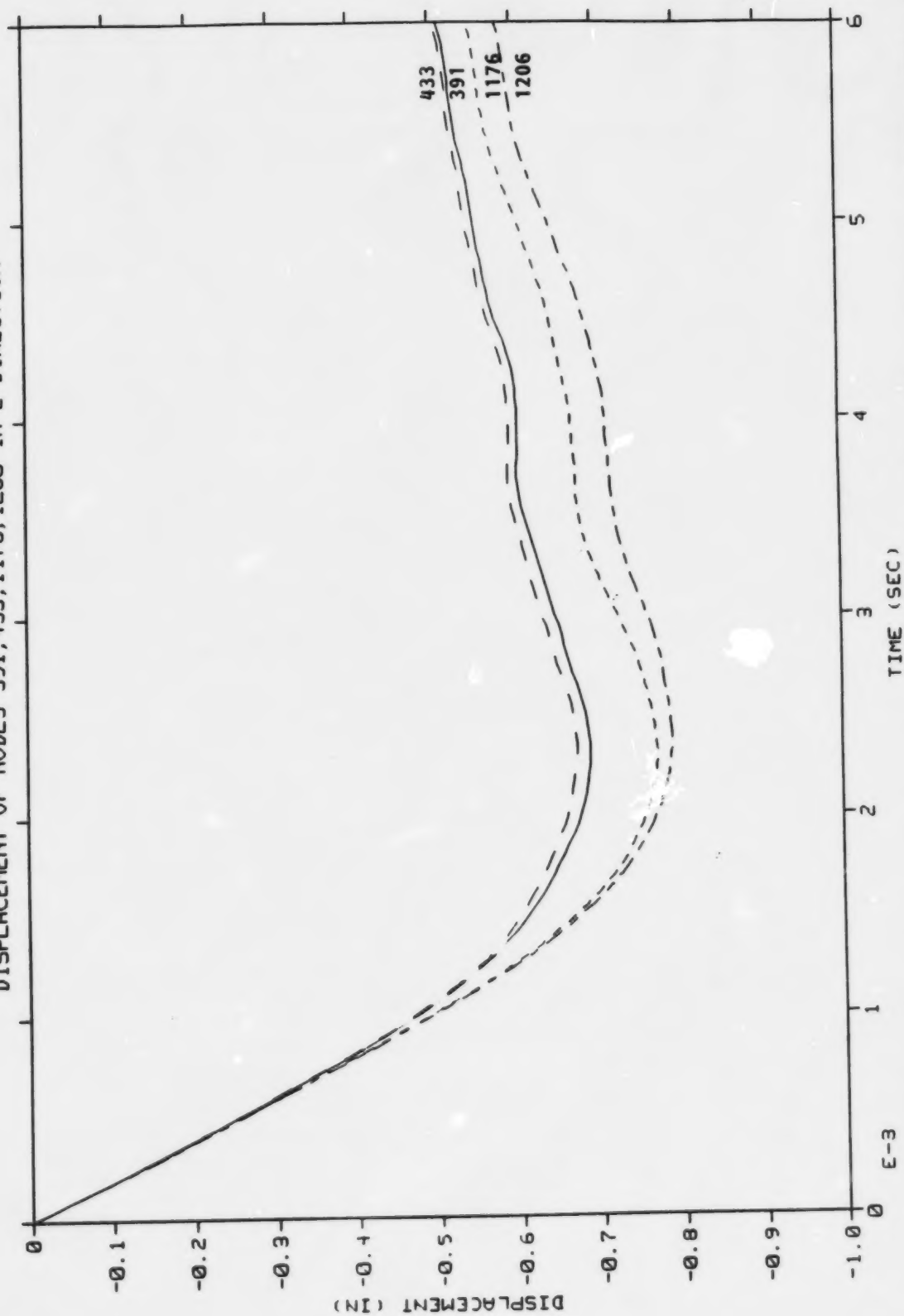


FIGURE 5.2.7

THUTP 9M BOTTOM DOWN DROP F.S ANALYSIS

DISPLACEMENT OF NODES 391, 433, 1176, 1206 IN Z-DIRECTION

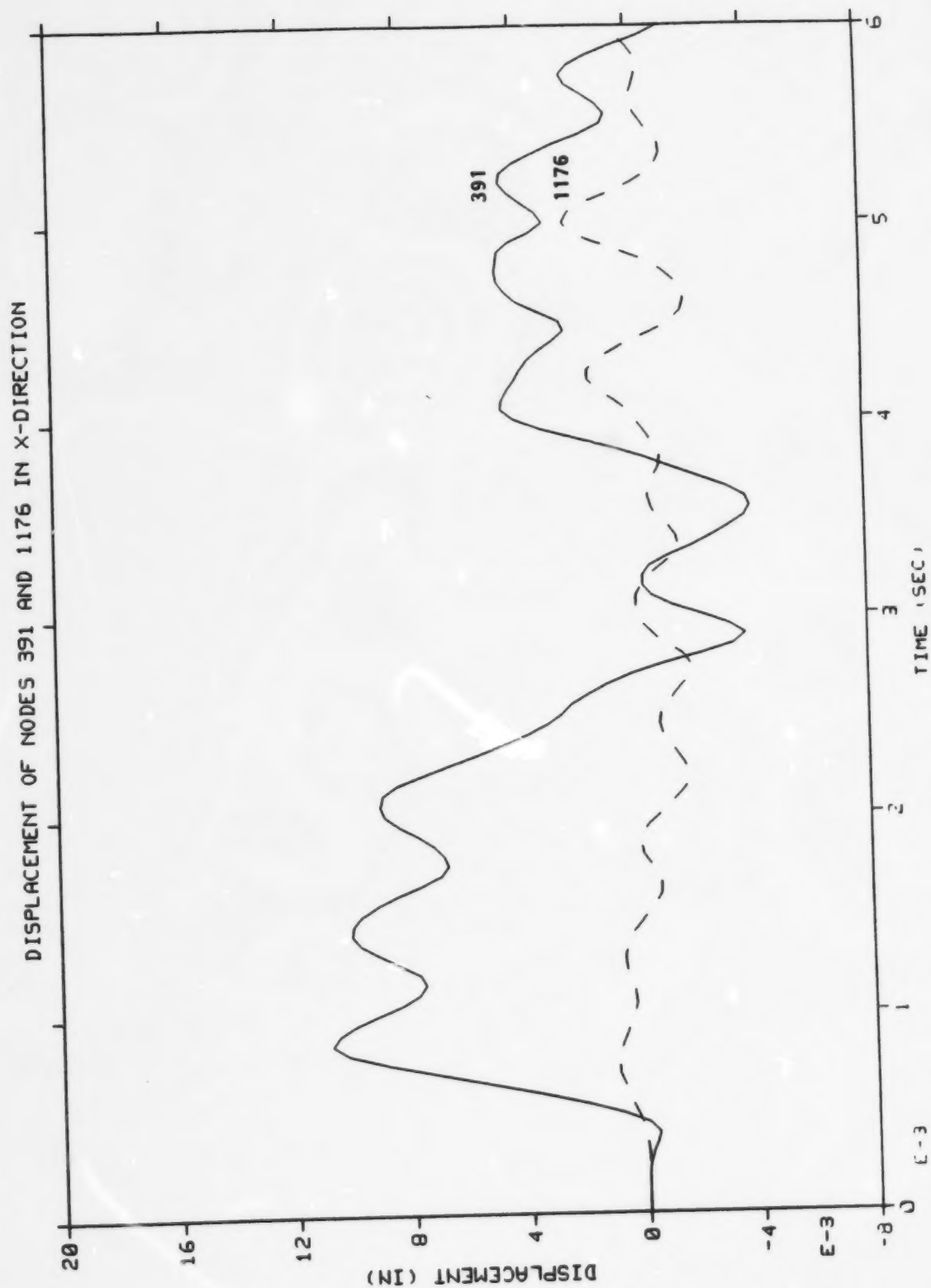


TIME (SEC)

DISPLACEMENT (IN)

0 E-3

THUTP 9M BOTTOM DOWN DROP F.S ANALYSIS



TIME (SEC)

FIGURE 5.2.9

THWTP 9M BOTTOM DOWN DROP F.S. ANALYSIS

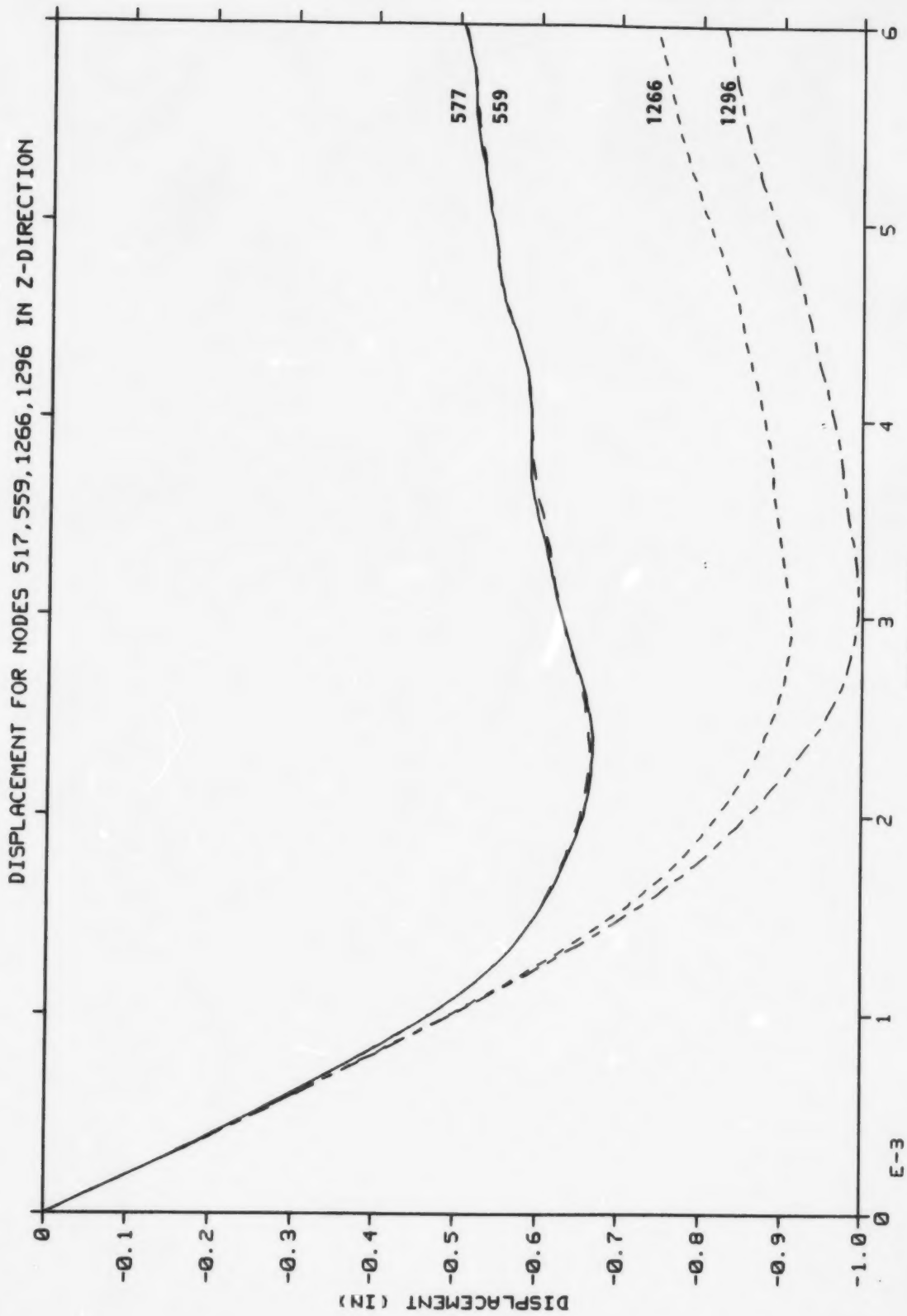


FIGURE 2

THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS

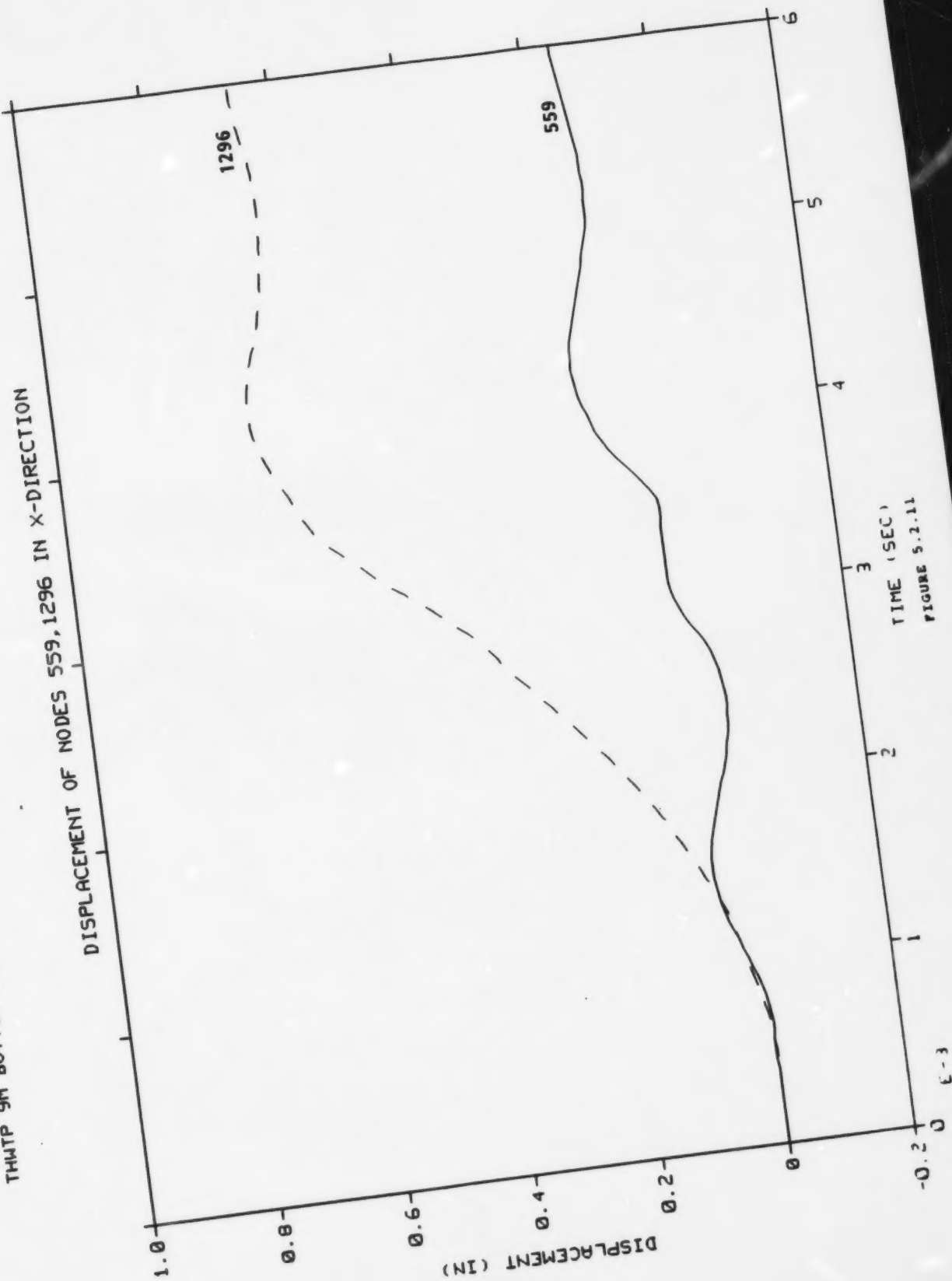
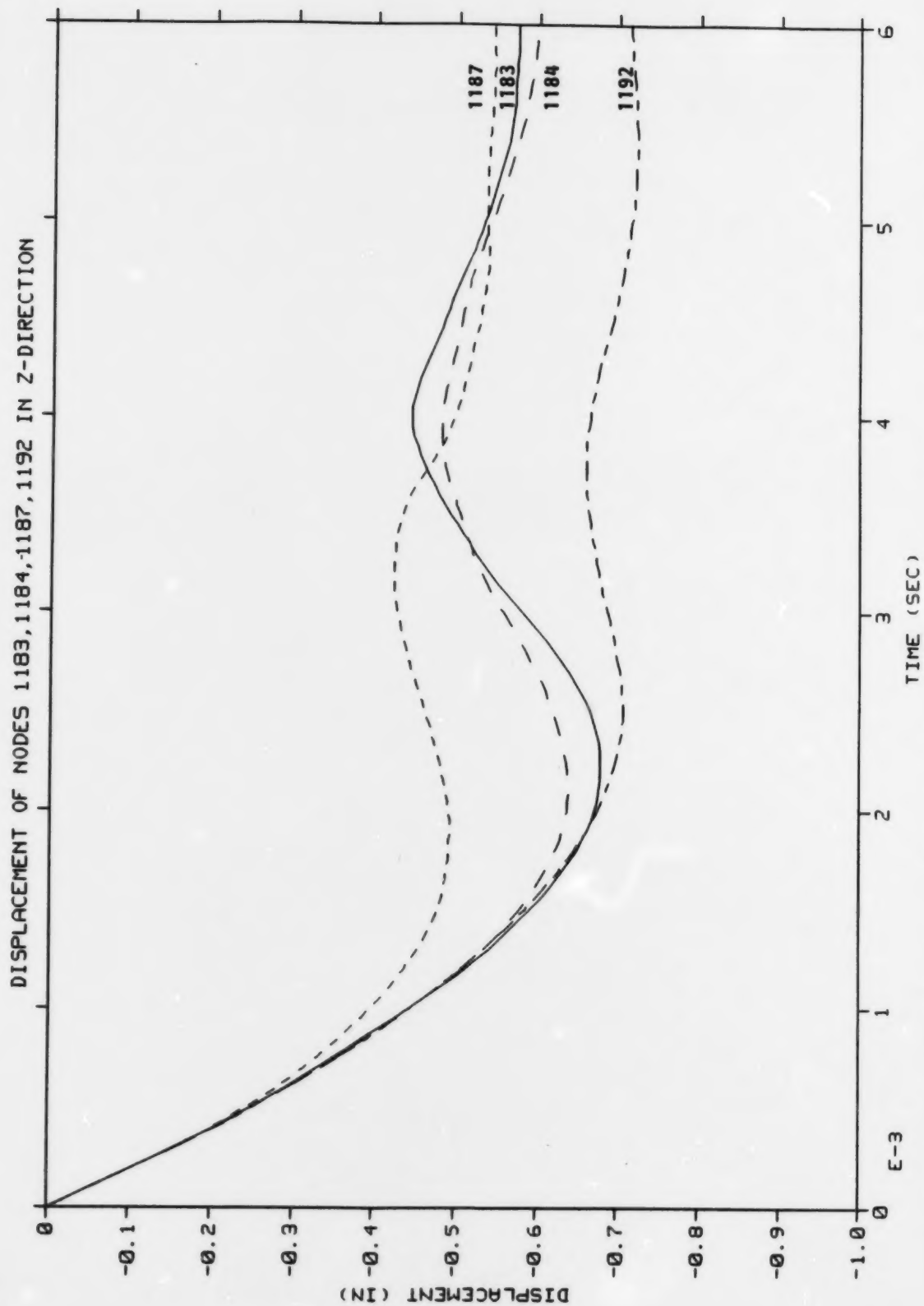


FIGURE 5.2.11

THWTP 9M BOTTOM DROP F.S ANALYSIS



THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS

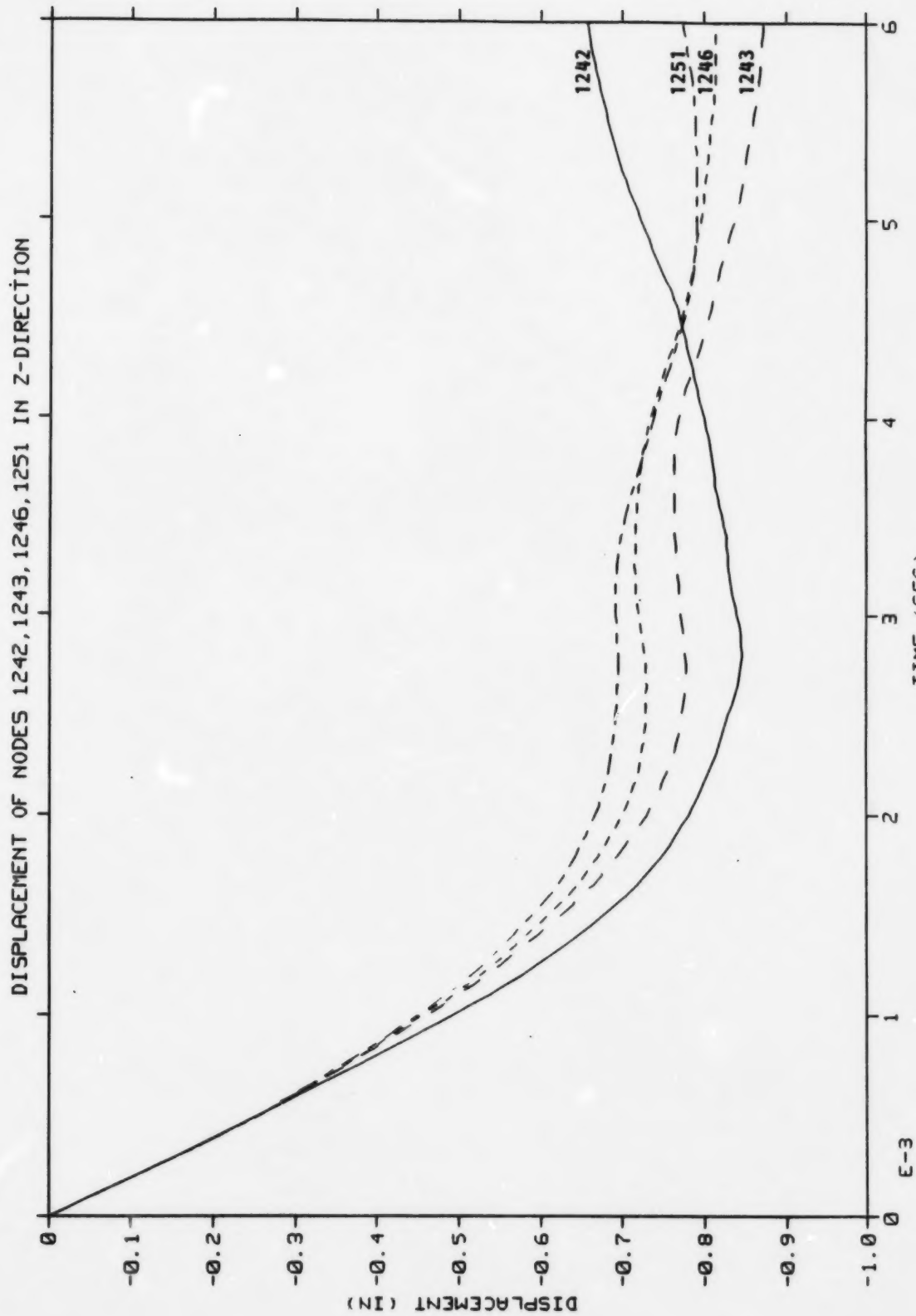
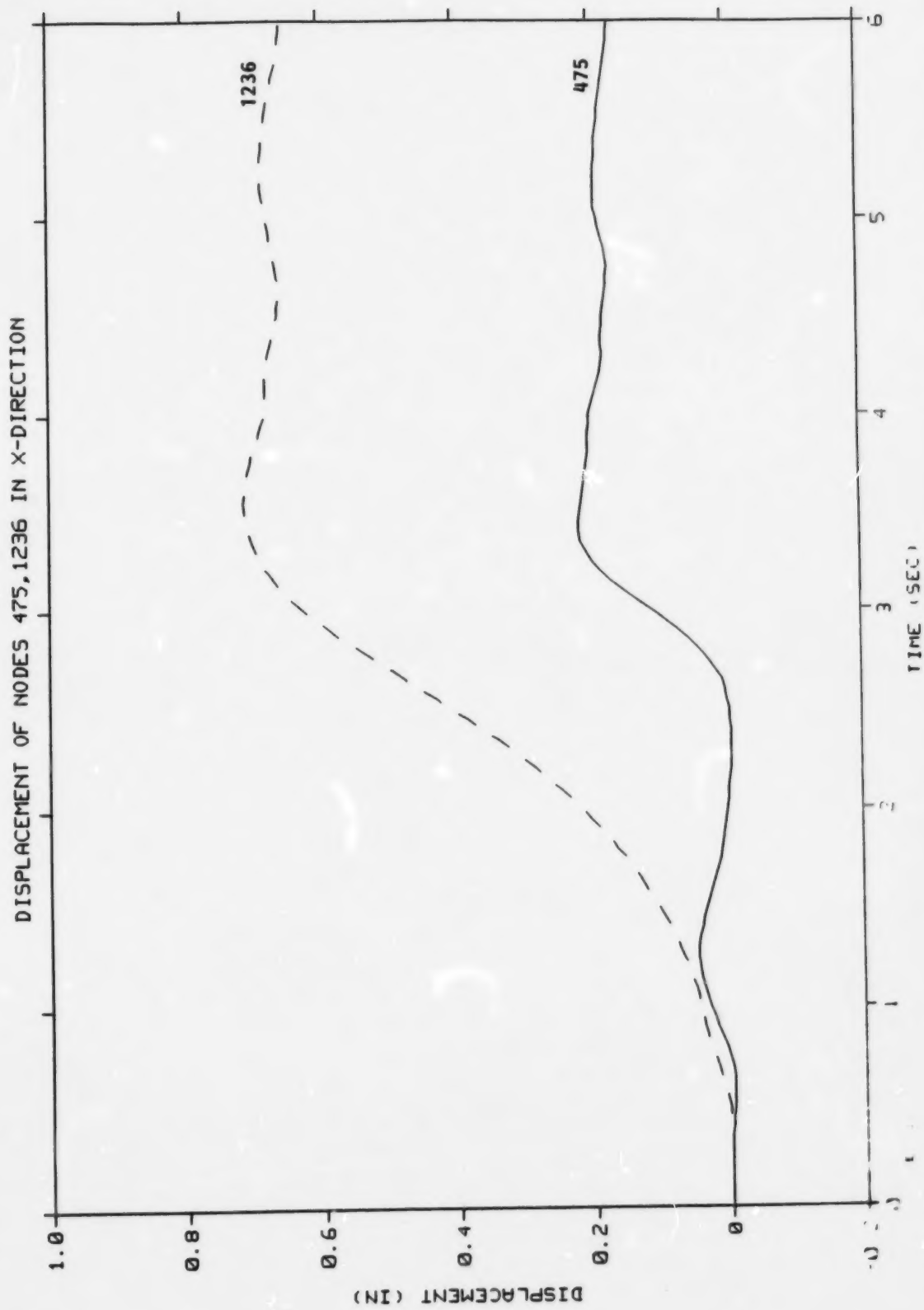


FIGURE 5.2.13

THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS



THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS

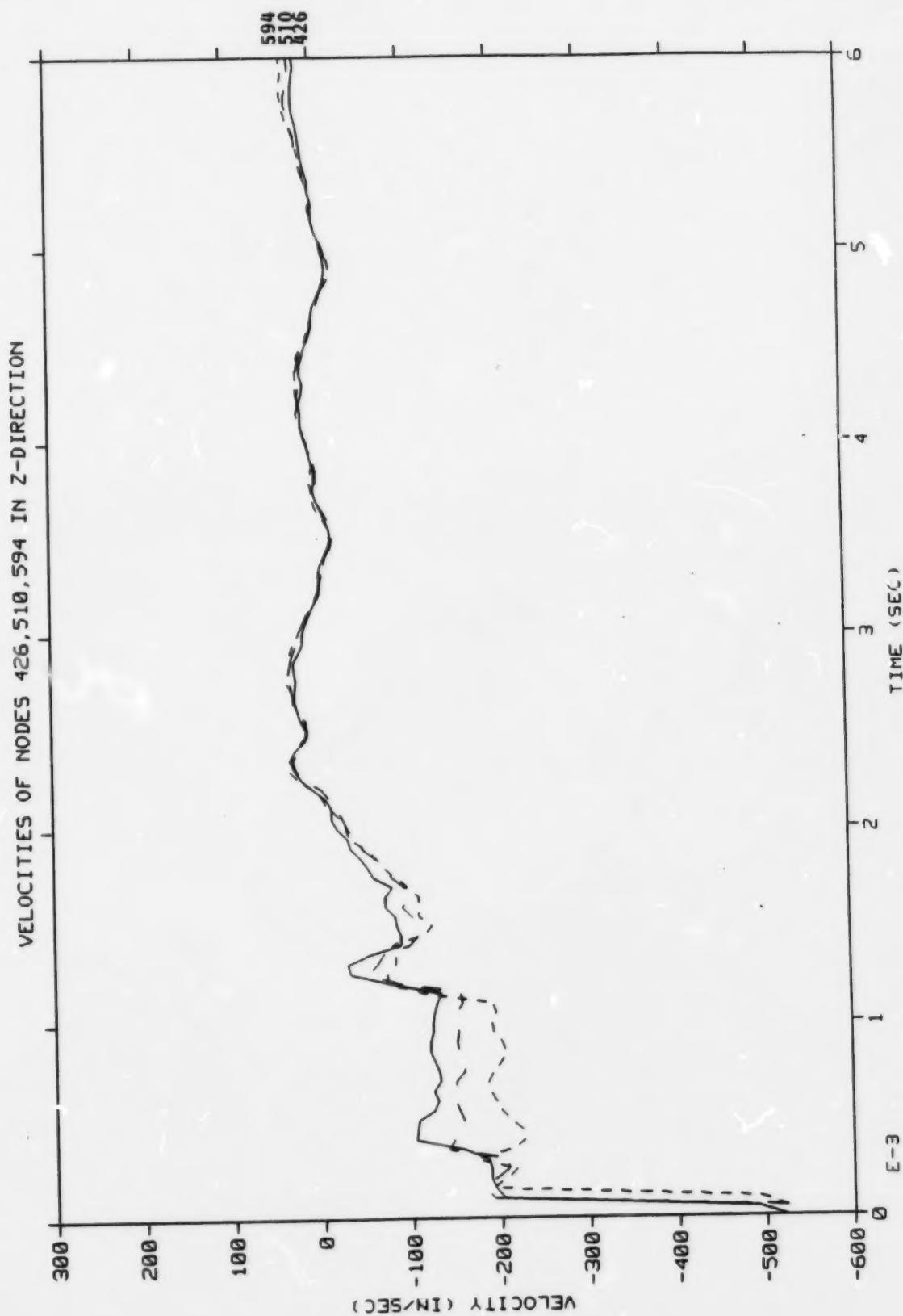
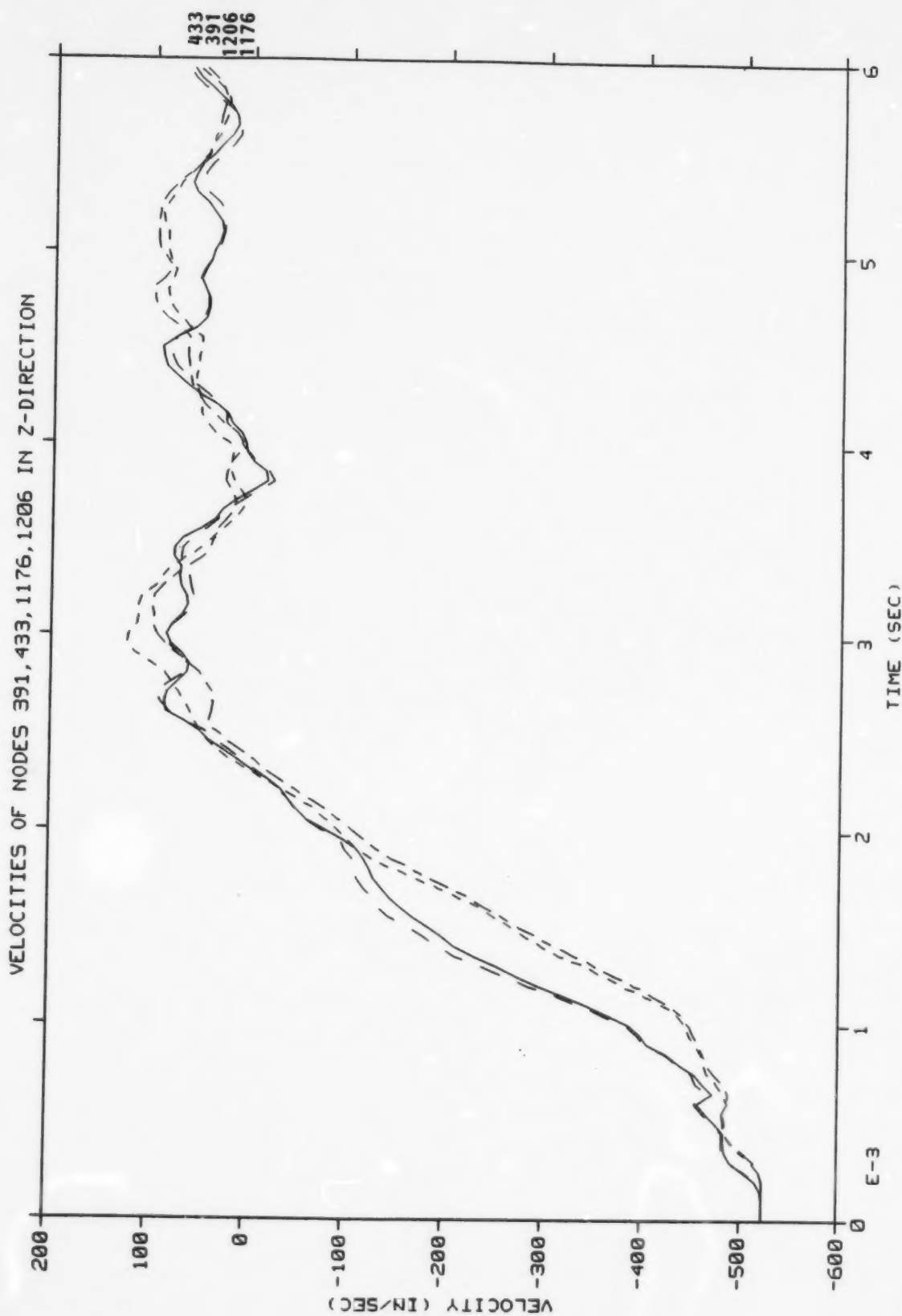


FIGURE 5.2.15

THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS



THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS

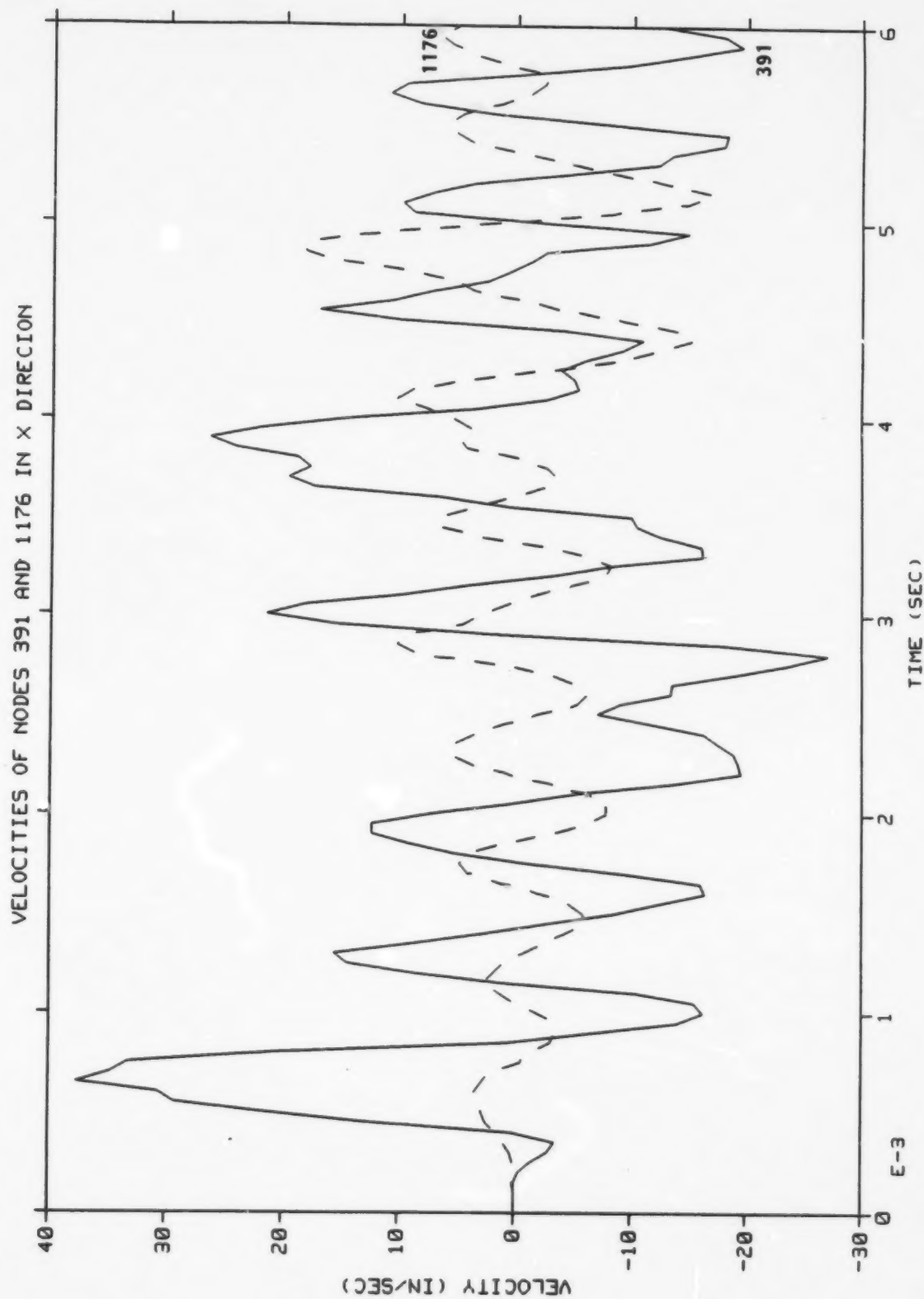
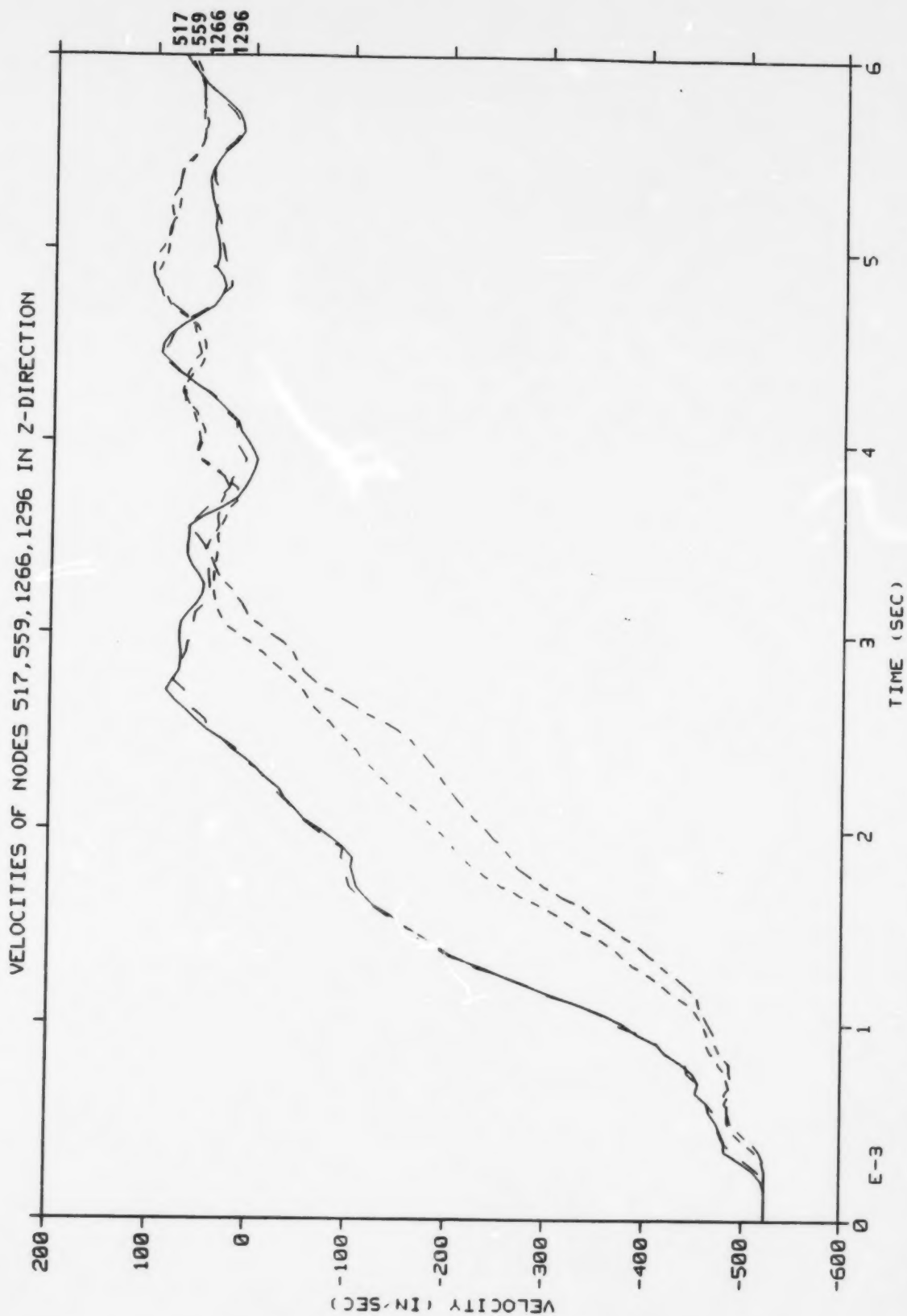


FIGURE 5.2.17

THWTP 9M BOTTOM DROP F.S ANALYSIS



THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS

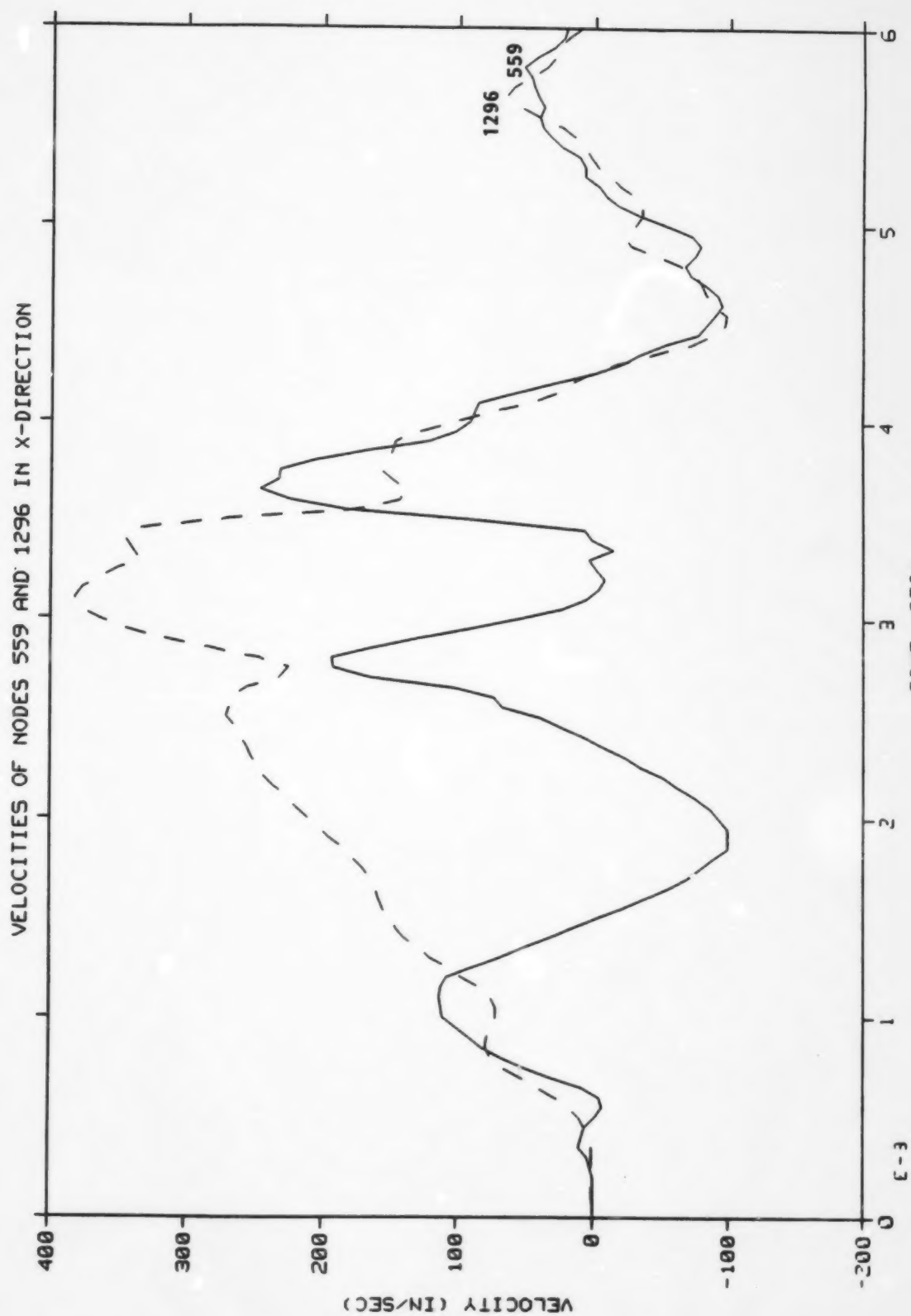
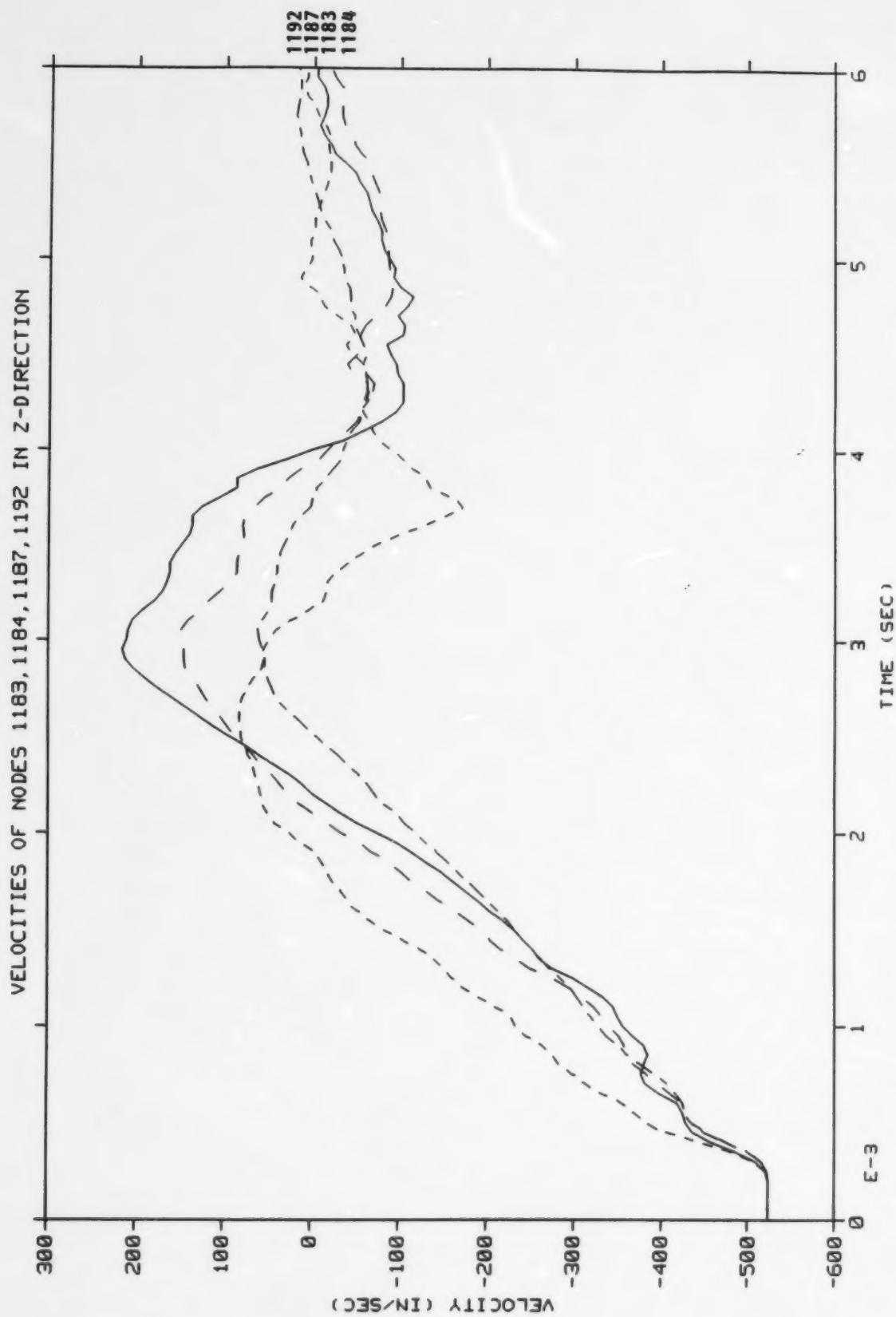


FIGURE 5.2.19

THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS



THMTP 9M BOTTOM DOWN DROP F.S ANALYSIS

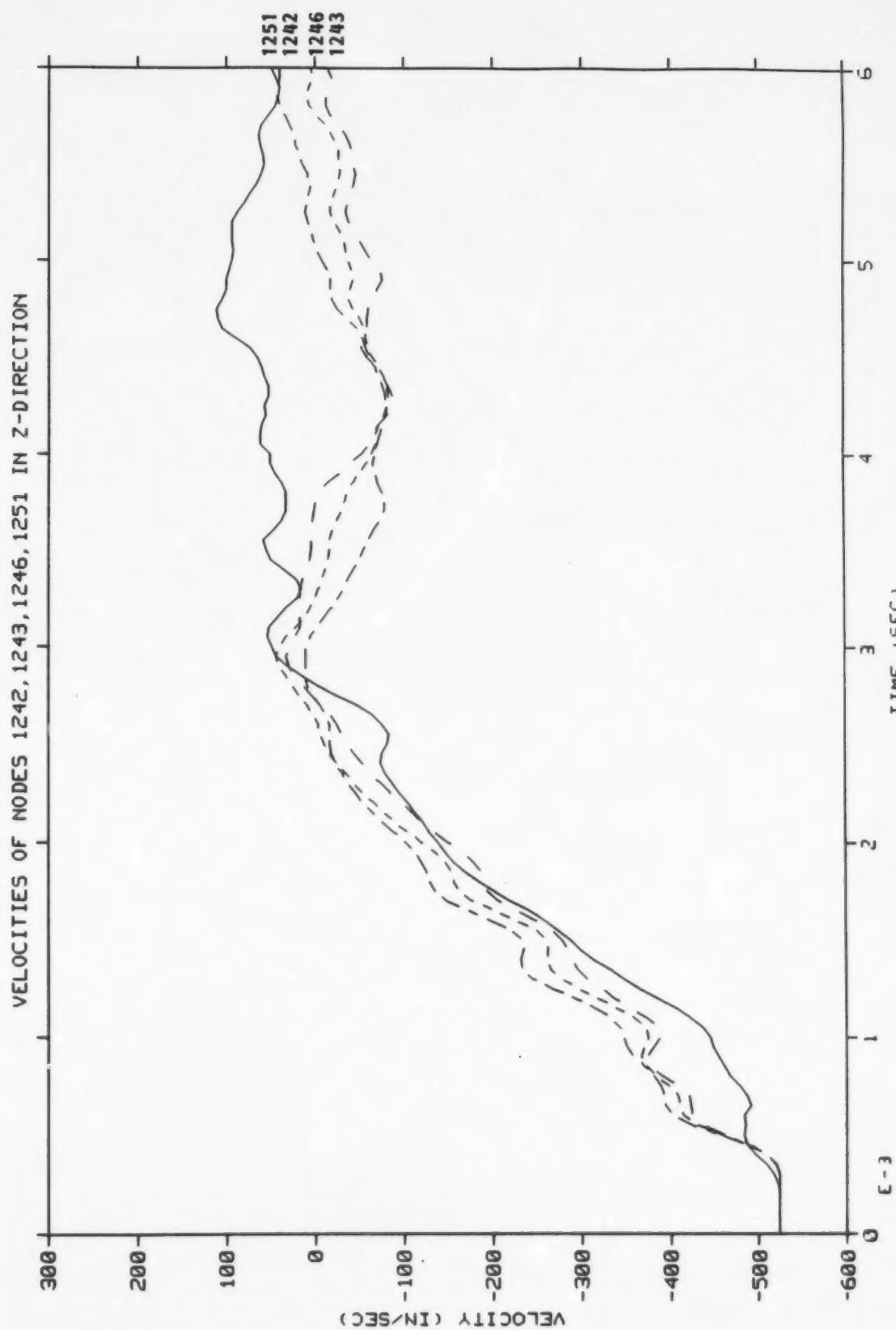
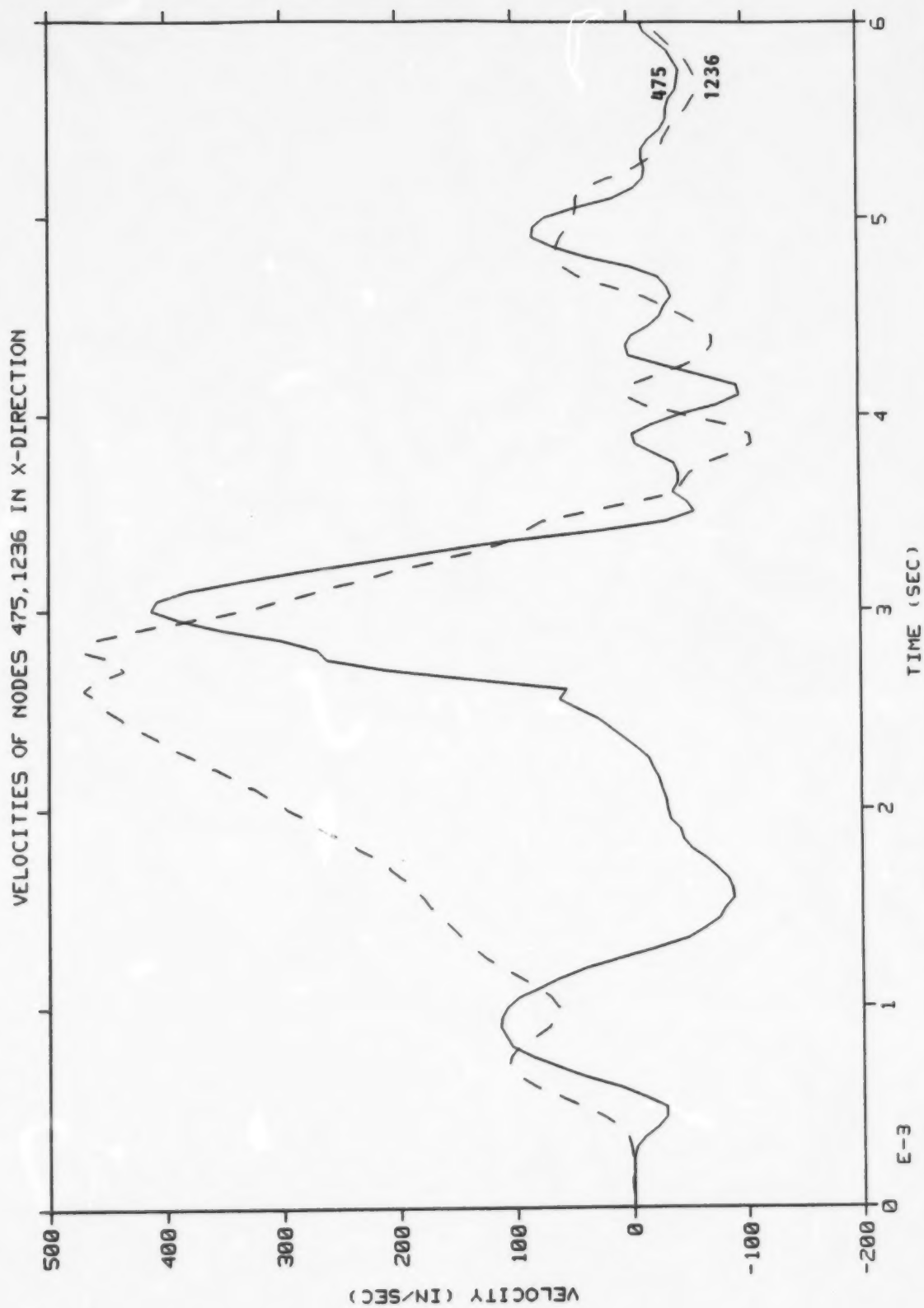


FIGURE 5.2.21

THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS



THMTP 9M BOTTOM DOWN DROP F.S ANALYSIS

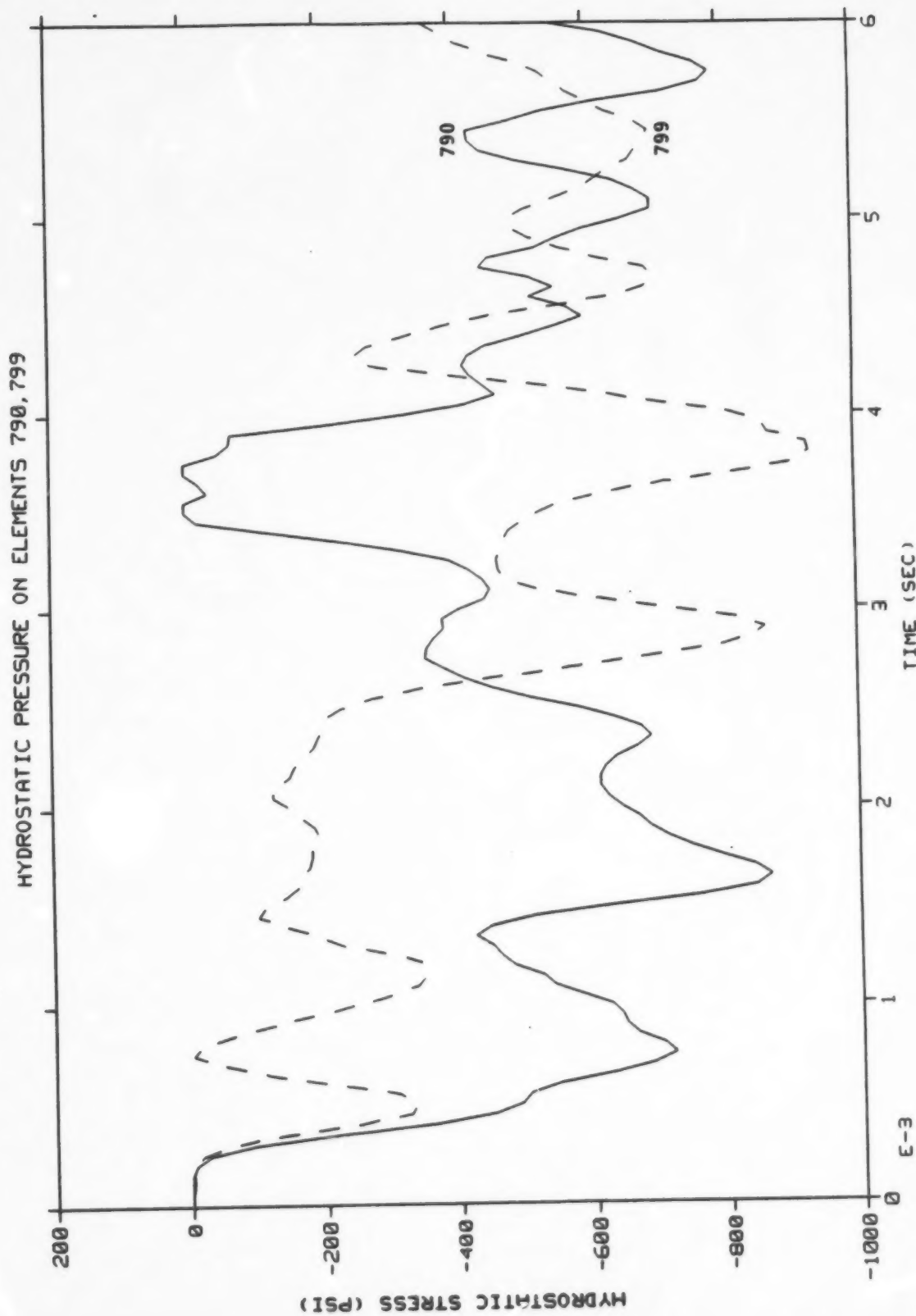
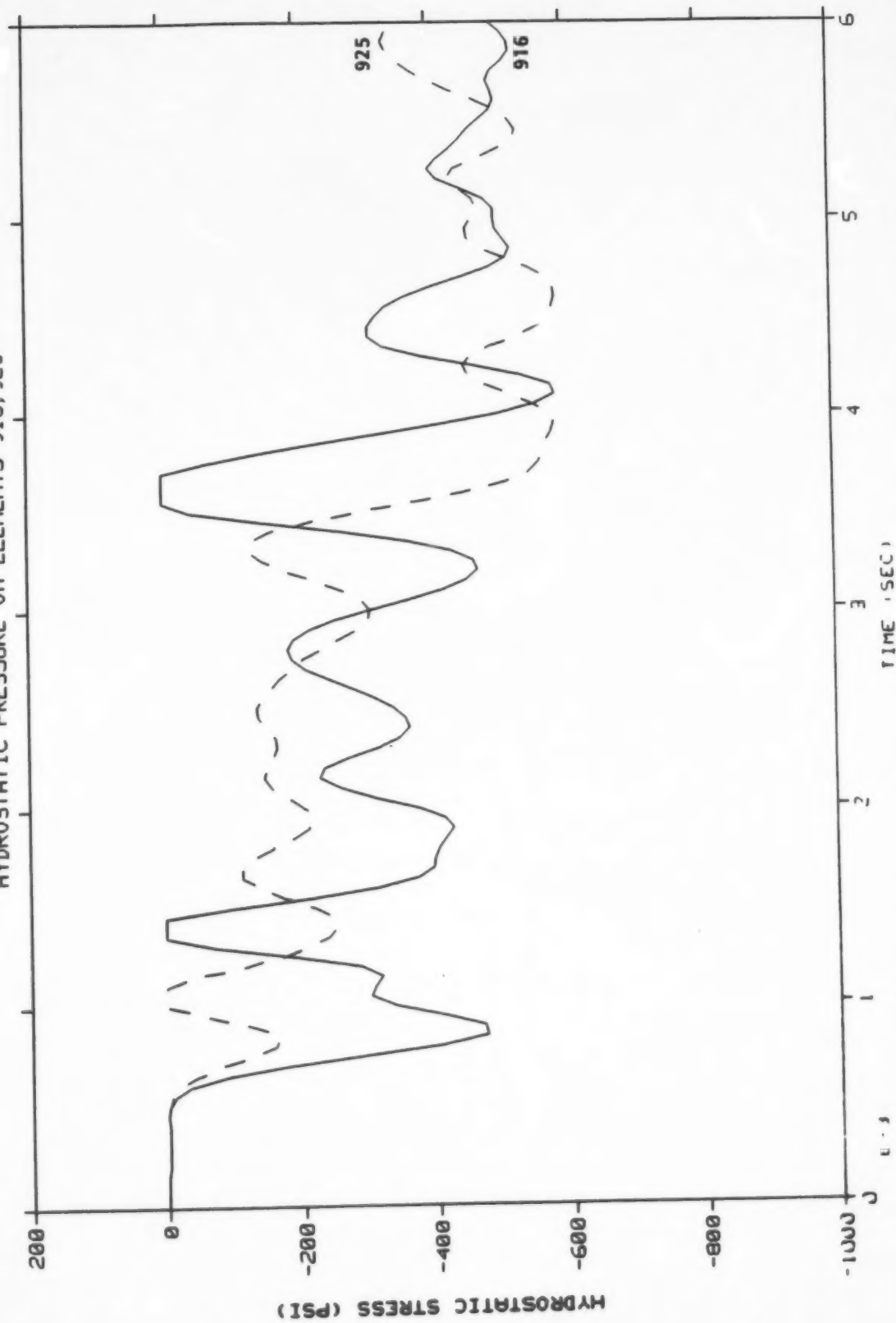


FIGURE 5.2.23

THWTP 9M BOTTOM DOWN DROP F.S. ANALYSIS

HYDROSTATIC PRESSURE ON ELEMENTS 916, 925



THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS

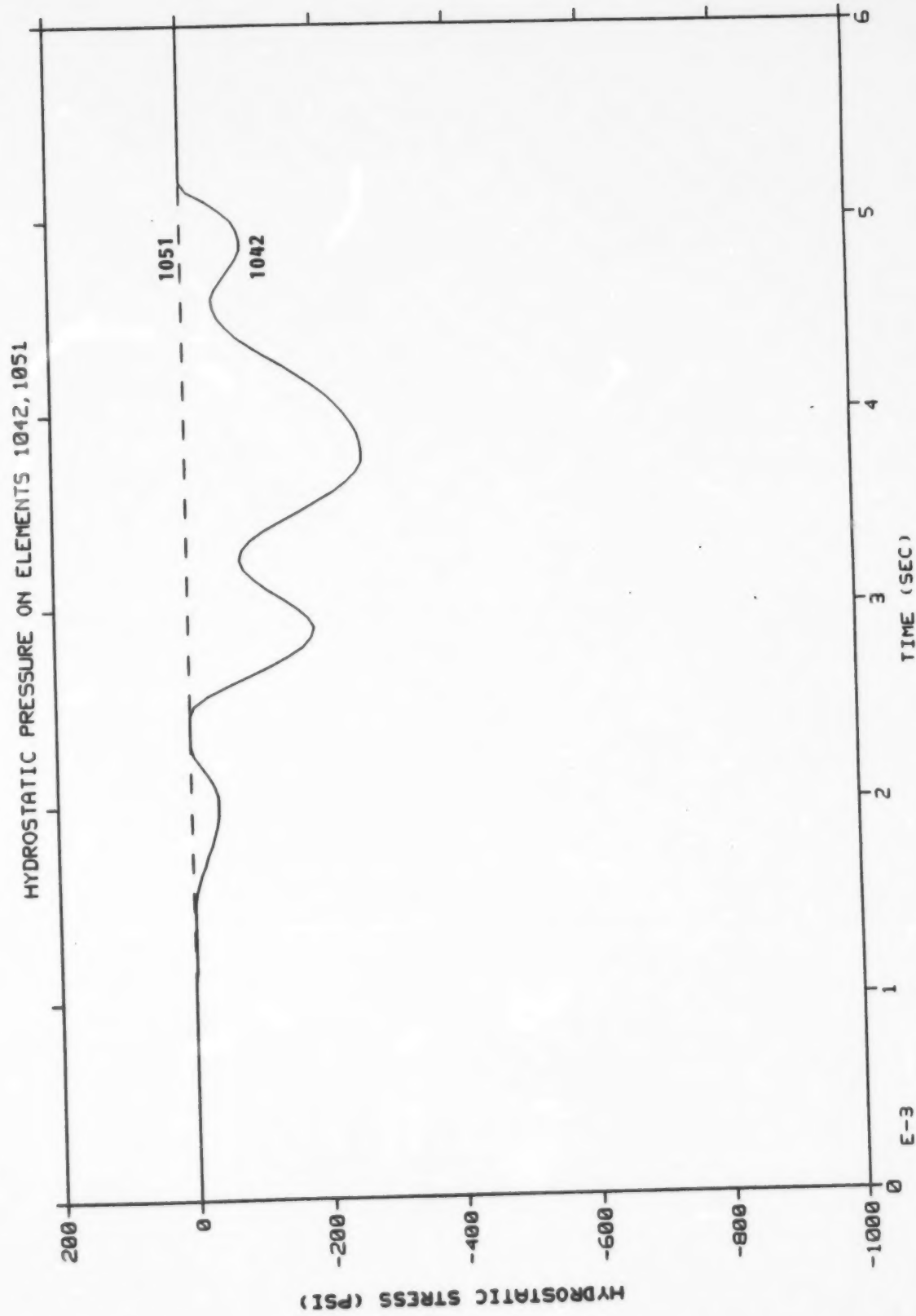


FIGURE 5.2.25

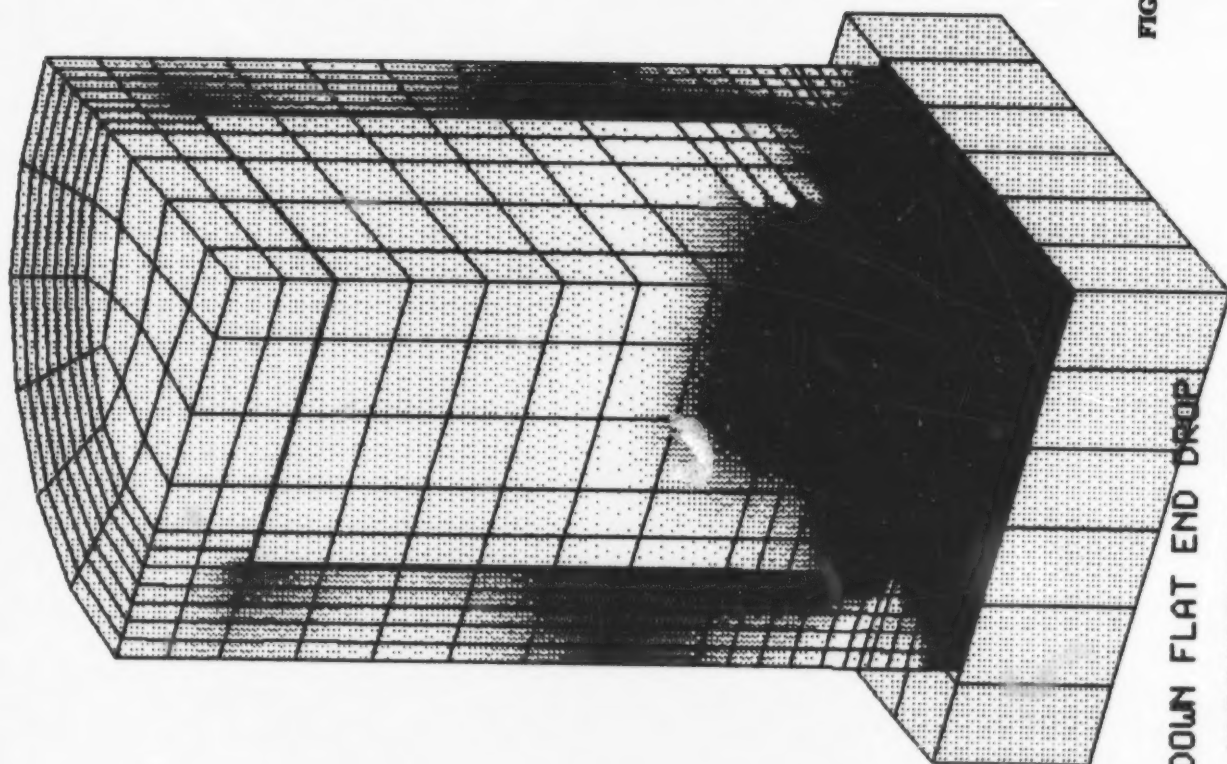
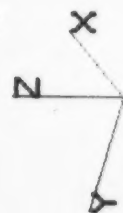


FIGURE 5.2.26

THUJTP FSI 9M BOTTOM DOWN FLAT END DROP
TIME=0.00099 S

HYDROSTATIC STRESS



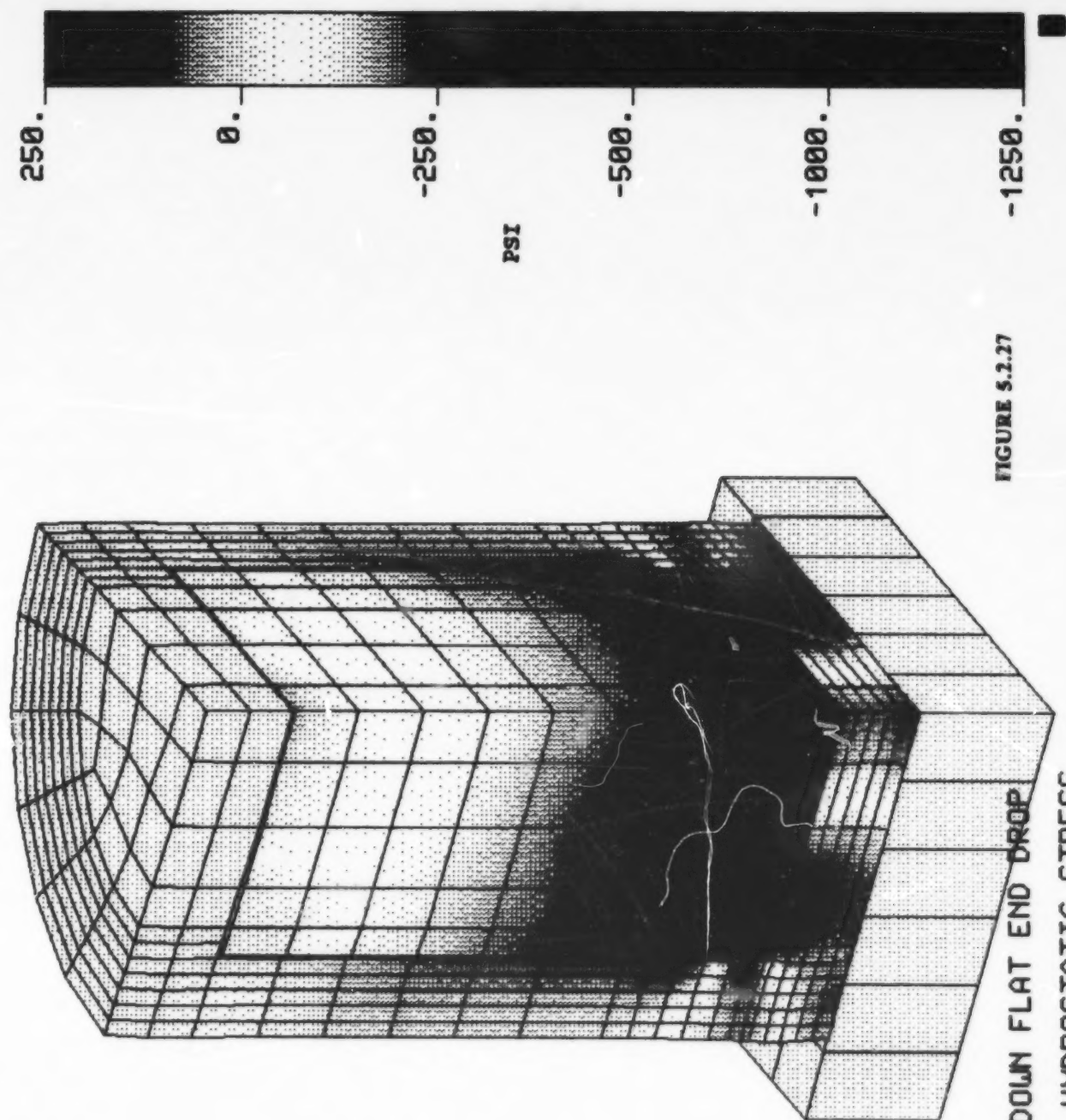


FIGURE S.2.27

FSI 9M BOTTOM DOWN FLAT END DROP
0.00299 S
HYDROSTATIC STRESS

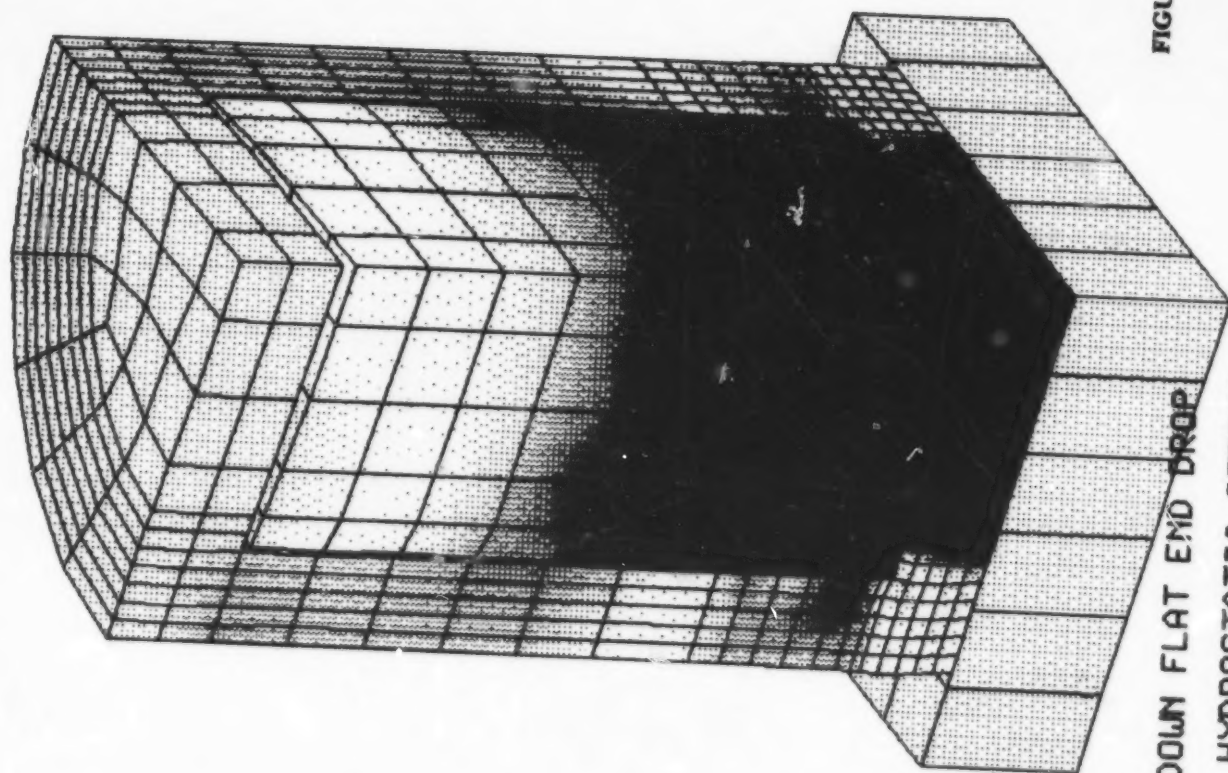
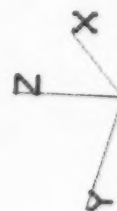
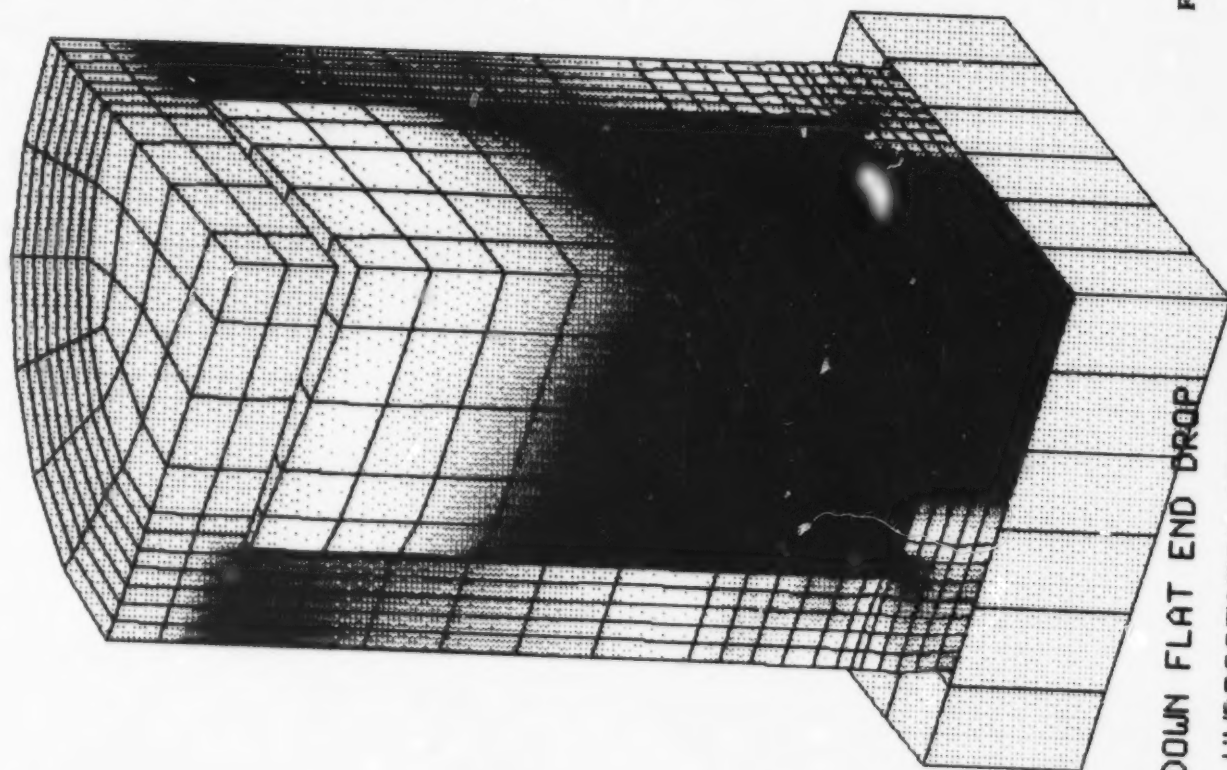


FIGURE 5.2.28

THWTP FSI 9M BOTTOM DOWN FLAT END DROP
TIME=0.00499 S
HYDROSTATIC STRESS





THWTP FSI 9M BOTTOM DOWN FLAT END DROP
 TIME=0.00599 S
 HYDROSTATIC STRESS

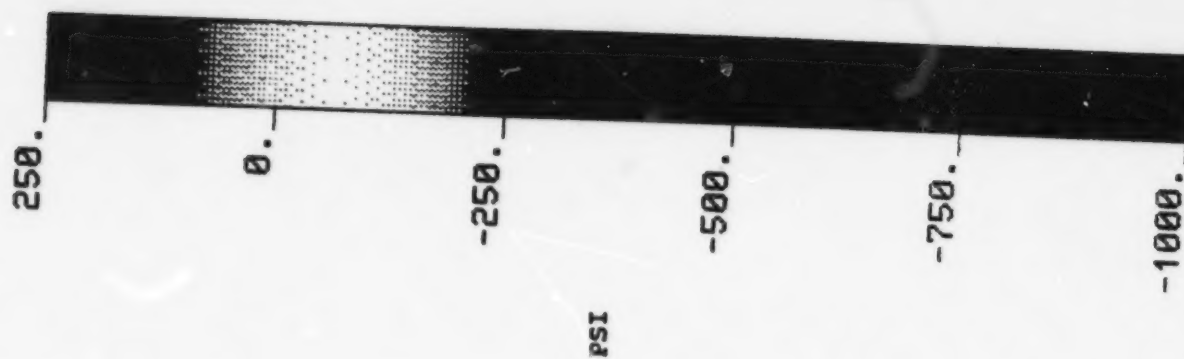
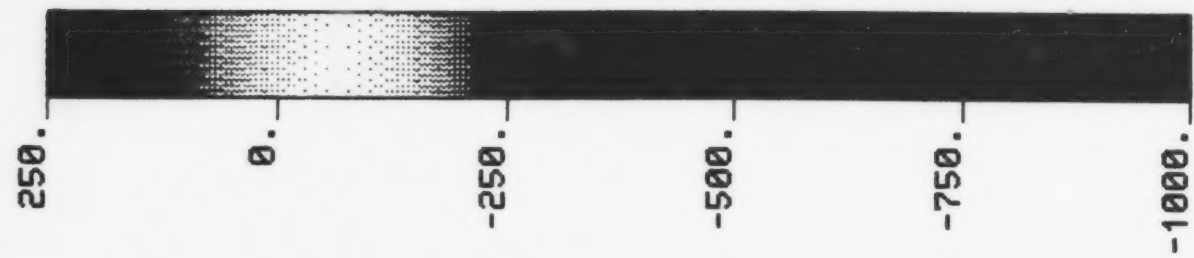
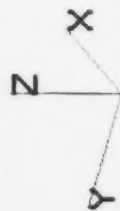
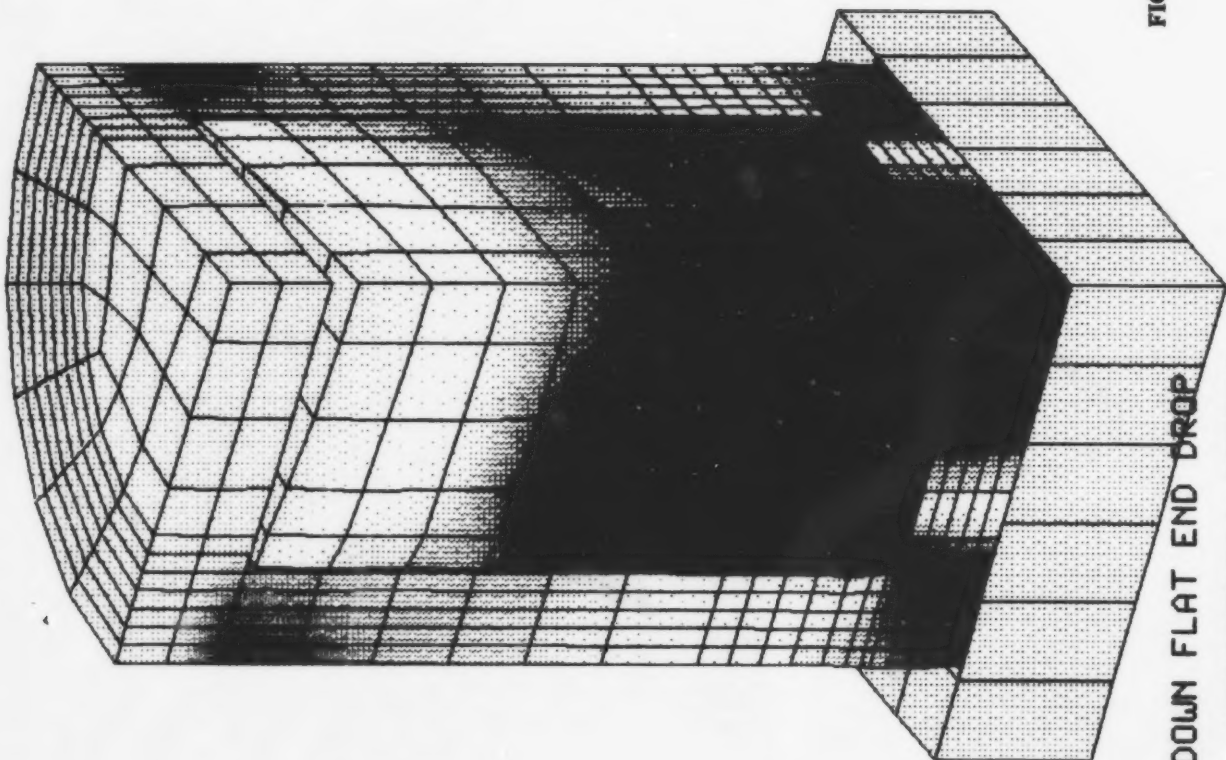


FIGURE 5.2.29



PSI

FIGURE 5.2.30



THJTP_FSI_9M_BOTTOM_DOWN_FLAT_END_DROP
TIME=0.00699 S

HYDROSTATIC STRESS

THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS

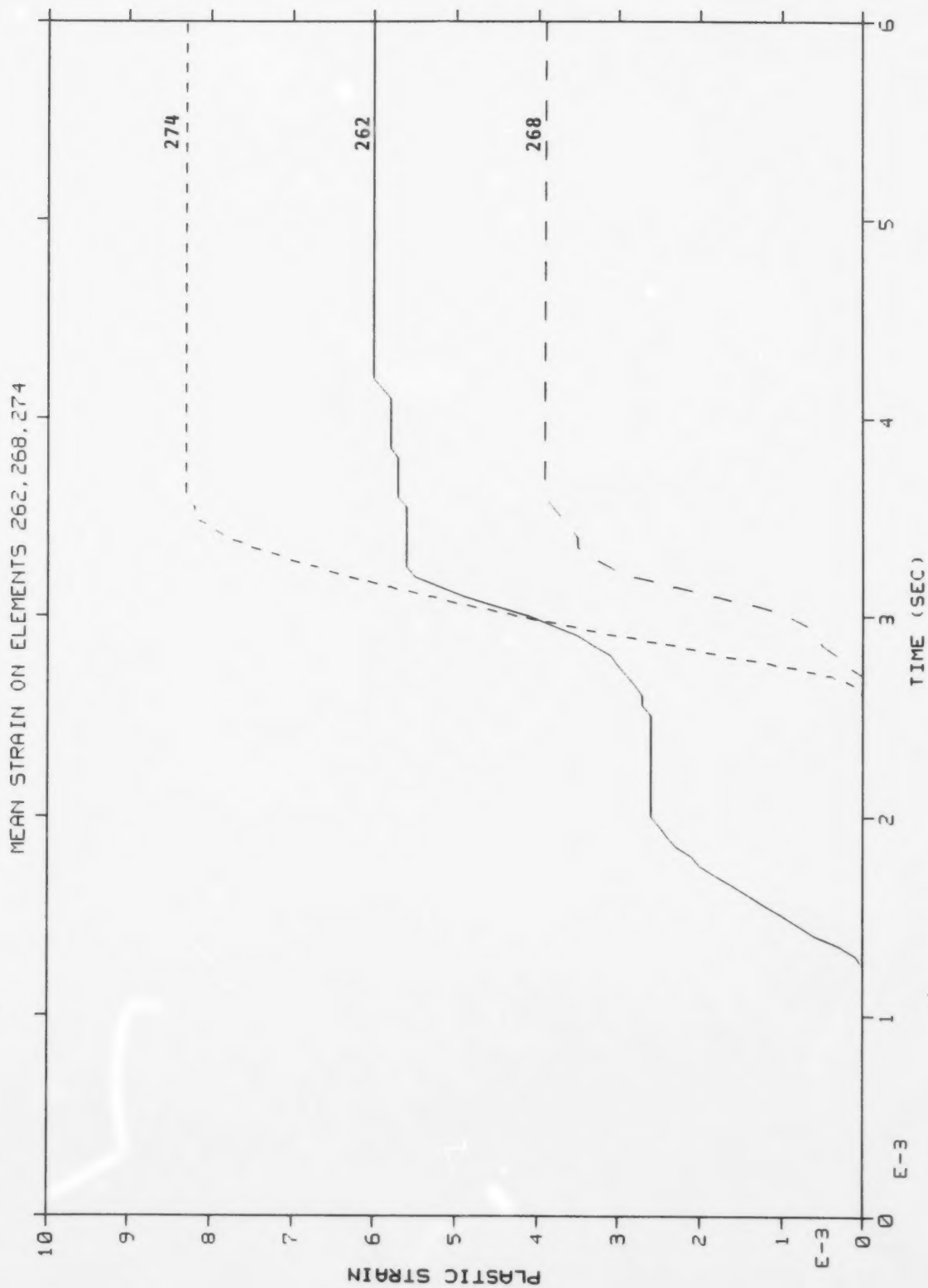


FIGURE 5.2.31

THKTP 9M BOTTOM DOWN DROP F.S ANALYSIS

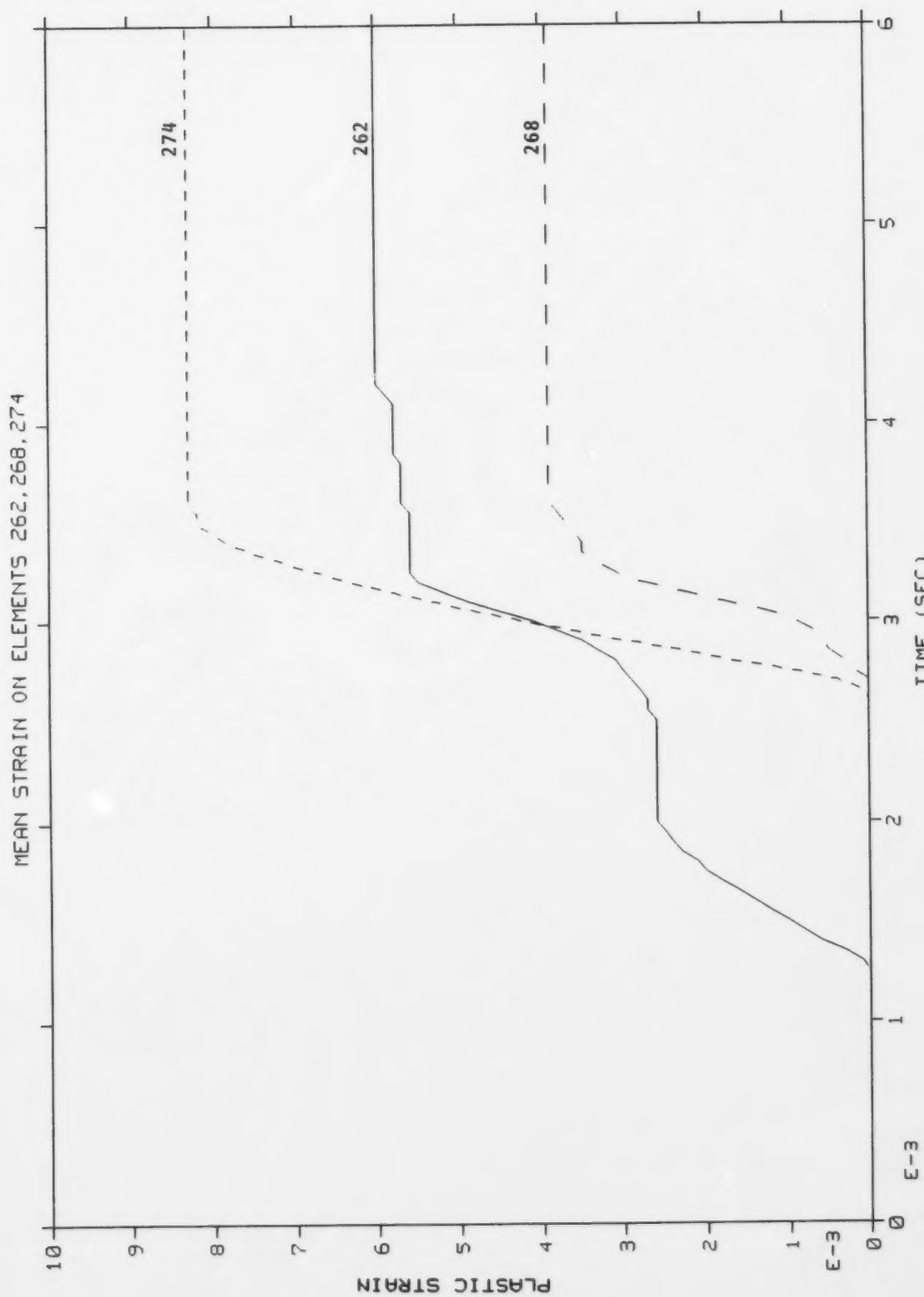


FIGURE 5.2.31

THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS

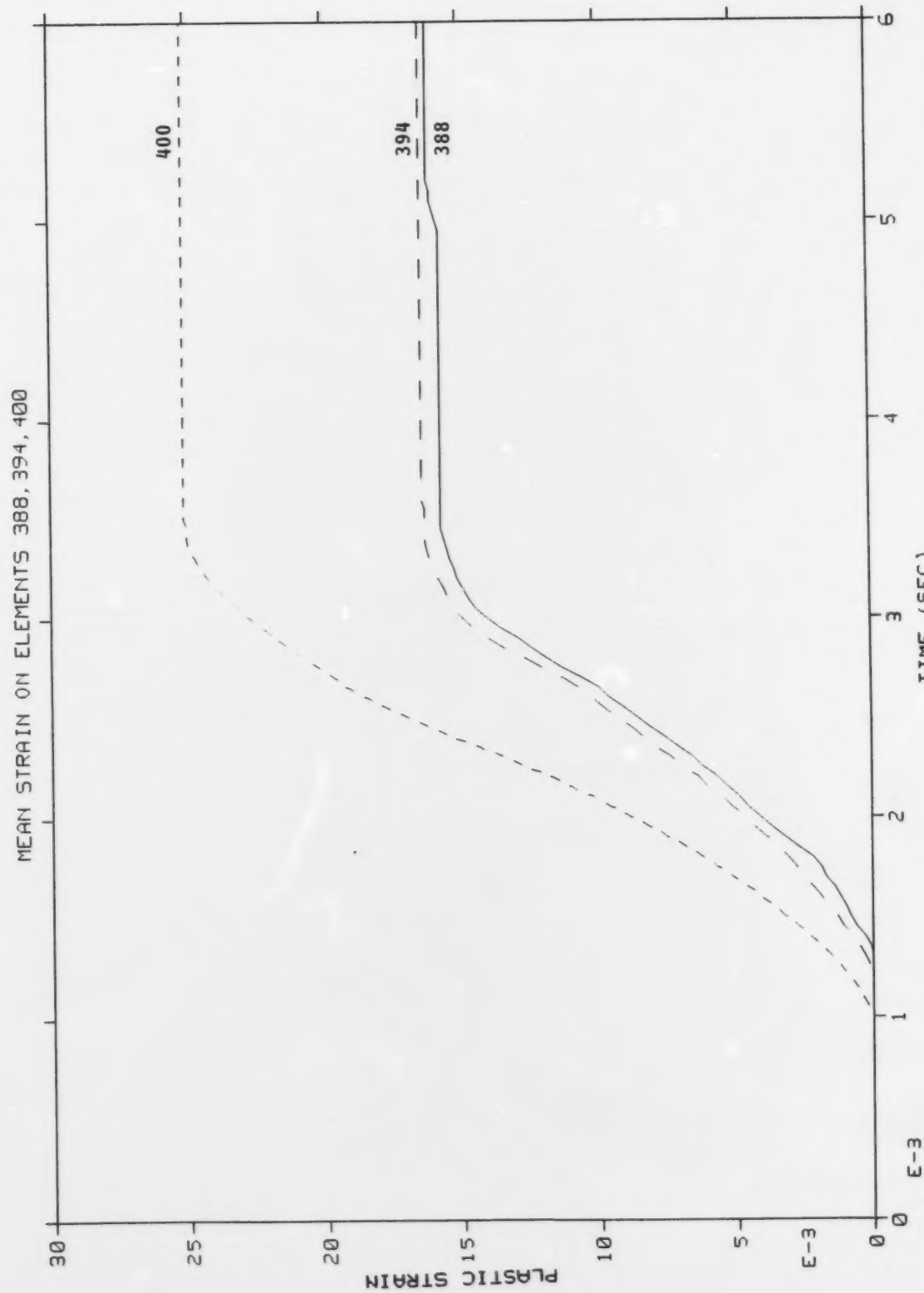
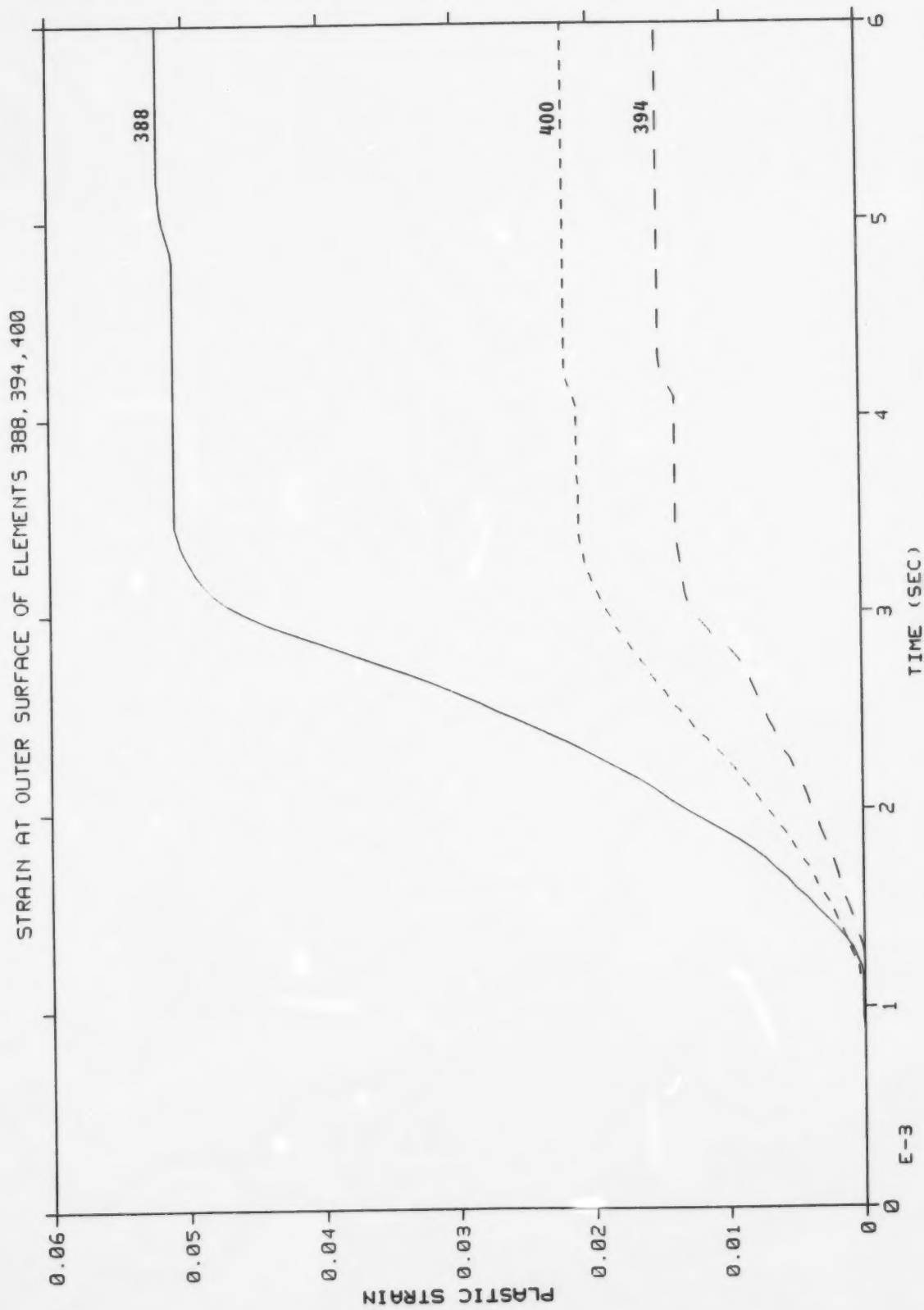


FIGURE 5.2.33

THWTP 9M BOTTOM DOWN DROP F.S ANALYSIS



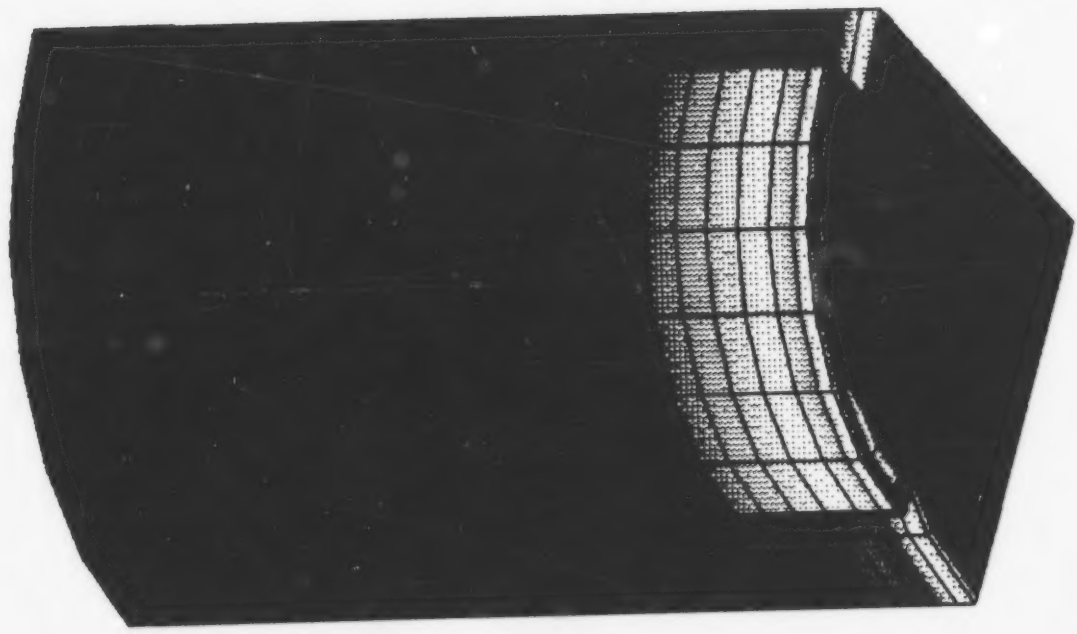
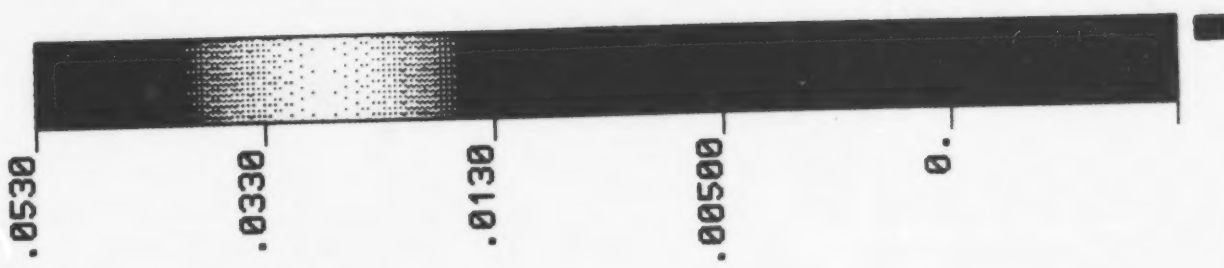


FIGURE 5.2.35

THWTP FSI 9M BOTTOM DOWN FLAT END DROP
TIME=0.00599 S
EFF. PLASTIC STRAIN

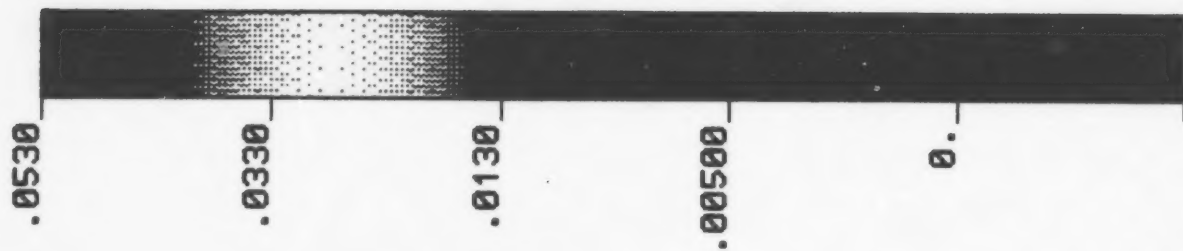


FIGURE 5.2.36

THWTP FSI 9M BOTTOM DOWN FLAT END DROP
 TIME=0.00599 S
 EFF. PLASTIC STRAIN

APPENDIX A1
HYDRODYNAMIC ELEMENT FORMULATION FOR
FLUID - STRUCTURE INTERACTION



DERIVATION OF A SIMPLIFIED HYDRODYNAMIC ELEMENT

FOR FLUID-STRUCTURE INTERACTION

A1.1 FLUID MOMENTUM EQUATIONS

Considering a differential volume and summing the forces acting on the fluid particle results in the following momentum differential equations:

$$\begin{aligned} \rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] &= F_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \\ \rho \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] &= F_y + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \\ \rho \left[\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] &= F_z + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \end{aligned}$$

where; ρ - mass density

u, v, w are primitive variables of velocity with respect to x, y and z directions respectively.

F_x, F_y, F_z are the body force components in the x, y and z directions respectively.

(i.e. under gravitational acceleration in the y - direction
 $F_x = F_z = 0, F_y = \rho g$)

The normal stresses $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ include the hydrostatic fluid pressure P as follows: (Newtonian fluids using Stokes law of friction)

$$\begin{aligned} \sigma_{xx} &= -P + \sigma'_x \\ \sigma_{yy} &= -P + \sigma'_y \\ \sigma_{zz} &= -P + \sigma'_z \end{aligned}$$

where the primes indicate deviatoric components.

The constitutive equations for a Newtonian fluid with the bulk viscosity set to zero (i.e. Stokes condition) are given by:

$$\begin{aligned}\tau_{xy} &= \tau_{yx} = \mu \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right] \\ \tau_{yz} &= \tau_{zy} = \mu \left[\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right] \\ \tau_{zx} &= \tau_{xz} = \mu \left[\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right] \\ \sigma_x' &= 2\mu \left[\frac{\partial u}{\partial x} - \frac{1}{3} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] \right] \\ \sigma_y' &= 2\mu \left[\frac{\partial v}{\partial y} - \frac{1}{3} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] \right] \\ \sigma_z' &= 2\mu \left[\frac{\partial w}{\partial z} - \frac{1}{3} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] \right]\end{aligned}$$

where μ is the molecular viscosity,

The continuity equation is derived from the conservation of mass:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

A1.3 ASSUMPTIONS

- i) An inviscid fluid is considered, thus $\mu = 0$ and:

$$\begin{aligned}\sigma_{xx} &= -P \\ \sigma_{yy} &= -P \\ \sigma_{zz} &= -P\end{aligned}$$

- ii) For relatively small disturbances the convective (mass transport) terms may be neglected.

Thus;

$$\rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] = 0$$

, etc.

In other words, the material derivative is approximated by the partial derivative with respect to time.

- iii) Body forces F_x , F_y , F_z are neglected (i.e. small by comparison to dynamic forces).

A1.4 REDUCED EQUATIONS

The reduced set of equations to be evaluated are:

$$\begin{aligned}\rho \frac{\partial u}{\partial t} &= - \frac{\partial \sigma_{xx}}{\partial x} \\ \rho \frac{\partial v}{\partial t} &= - \frac{\partial \sigma_{yy}}{\partial y} \\ \rho \frac{\partial w}{\partial t} &= - \frac{\partial \sigma_{zz}}{\partial z}\end{aligned}$$

where;

$$\begin{aligned}\sigma_{xx} &= -P \\ \sigma_{yy} &= -P \\ \sigma_{zz} &= -P\end{aligned}$$

In matrix form:

$$\rho \frac{\partial (u)}{\partial t} = \begin{Bmatrix} - \frac{\partial P}{\partial x} \\ - \frac{\partial P}{\partial y} \\ - \frac{\partial P}{\partial z} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

A1.5 EQUATION OF STATE:

The thermodynamic pressure, P , is obtained using an equation of state;

$$P = f(\rho, T)$$

where ρ is density and T is absolute temperature. Neglecting friction leads to;

$$P = f(\rho)$$

where the thermodynamic pressure is equal to the mean pressure.

The form used for the fluid in this analysis is given by:

$$P = K \frac{\Delta \rho}{\rho_0}$$

where K is the bulk modulus and ρ_0 is the original density.

From the conservation of mass;

$$\rho_0 V_0 = \rho^t V^t$$

where V_0 = original volume

ρ^t, V^t = density, volume at time 't'

$$\begin{aligned} \text{Therefore } P^t &= K \left[\frac{\rho^t - \rho_0}{\rho_0} \right] \\ &= \frac{K}{\rho_0} \left[\frac{\rho_0 V_0}{V^t} - \rho_0 \right] \\ P^t &= K \left[\frac{V_0}{V^t} - 1 \right] \end{aligned}$$

The above form is used to characterize the fluid. Different relations can be used for other applications.

A1.6 DISCRETIZATION OF FLOW EQUATIONS:

Considering 1-dimensional flow, the equations reduce to:

$$\rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial x} = 0$$

Applying Galerkin's Method (Integral or Weak form of the equilibrium equations):

$$R_L = L\bar{u} - f$$

where $\bar{u} = u$ and u is the exact solution.

Select weight functions $N_i = N_i(x)$ and set the weighted average of the residual $R_L = 0$.

Considering a 2-node linear shape function;

$$\bar{u} = Nd = \sum_{i=1}^2 N_i d_i$$

where d_i are the nodal values of velocity u .

$$R_i = \int_V N_i(x) R_L dv = 0$$

(Note: Surface traction boundary conditions are ignored).

Assuming a unit cross sectional area;

$$R_i = \int_0^L N_i \left(\rho \frac{\partial \bar{u}}{\partial t} + \frac{\partial P}{\partial x} \right) dx = 0$$

$$\int_0^L N_i \rho \frac{\partial \bar{u}}{\partial t} dx + \int_0^L N_i \frac{\partial P}{\partial x} dx = 0$$

Integrating the second term by parts:

$$\int u dv = uv - \int v du$$

$$u = N_i \quad du = \frac{\partial N_i}{\partial x} dx$$

$$dv = \frac{\partial P}{\partial x} dx \quad v = P$$

$$\int_0^L N_i \frac{\partial P}{\partial x} dx = \left[N_i P \right]_0^L - \int_0^L P \frac{\partial N_i}{\partial x} dx$$

$$\text{Substituting for } \bar{u} = Nd = \sum_{i=1}^2 N_i d_i$$

$$\sum_{j=1}^2 \int_0^L \rho N_i N_j \frac{\partial d_j}{\partial t} dx - \int_0^L \frac{\partial N_i}{\partial x} P dx = - \left[N_i P \right]_0^L$$

for $i = 1, 2$.

In matrix form (at the element level);

$$\underline{M \dot{d} + F_{int} - F_{ext}}$$

where;
$$\underline{M} = \frac{\rho \int_0^L N^T N dx}{}$$

$$\underline{F_{int}} = \frac{- \int_0^L \frac{\partial N^T}{\partial x} P dx - \int_0^L B^T P dx}{}$$

$$B = \frac{(1 \times 2)}{- \frac{\partial N}{\partial x}}$$

$$\underline{F_{ext}} = \left\{ \begin{array}{c} P_1 \\ -P_2 \end{array} \right\}$$

If instead of N_i , the following definition is used;

$$\delta \bar{u} = \sum_{i=1}^2 \delta d_i N_i$$

Then,

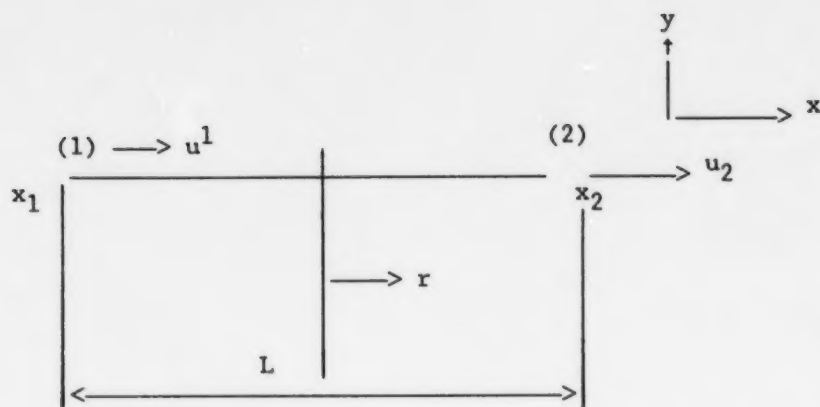
$$\rho \int_0^L N_i N_j \frac{\partial d_j}{\partial t} dx \delta d_i - \int_0^L \frac{\partial N_i}{\partial x} P dx \delta d_i = - \int_0^L N_i P \delta d_i$$

which is a statement of the virtual work principle, and, as with the Galerkin method, is a "weak form" of the equilibrium equations.

A1.7 SOLUTION IN 1-DIMENSIONAL SPACE

A1.7.1 Development of 1-D Element

Considering a two-noded element, the isoparametric formulation is as follows:



and linear shape functions;

$$N_1 = 1/2 (1 - r)$$

$$N_2 = 1/2 (1 + r)$$

$$x = \sum_{i=1}^2 N_i x_i$$

$$\frac{\partial N_1}{\partial r} = -0.5$$

$$\frac{\partial N_2}{\partial r} = 0.5$$

$$|J| = \left| \sum_{i=1}^2 \frac{\partial N_i}{\partial r} x_i \right| = 0.5x_2 - 0.5x_1 = 0.5L \text{ (Jacobian)}$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = -Bd ; \quad d = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\frac{\partial u}{\partial r} = \frac{\partial u \partial x}{\partial x \partial r}$$

$$\text{Therefore, } \frac{\partial u}{\partial x} = \left[\frac{\partial x}{\partial r} \right]^{-1} \frac{\partial u}{\partial r} ; \text{ where } \left[\frac{\partial x}{\partial r} \right]^{-1} = J^{-1}$$

$$\text{and since } \frac{\partial u}{\partial r} = -0.5 u_1 + 0.5 u_2$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \begin{bmatrix} -1/L & 1/L \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$F_{int} = \int_0^L B^T P dx = \int_{-1}^{+1} B^T P |J| dr$$

$$= \int_{-1}^{+1} \begin{bmatrix} \frac{1}{L} \\ -\frac{1}{L} \end{bmatrix} P \cdot 0.5L dr$$

$$F_{int} = \begin{Bmatrix} P \\ -P \end{Bmatrix}$$

A1.7.2 Lumped Mass Matrix

In the solution considered herein, a lumped mass matrix is considered;

$$M = \begin{bmatrix} \rho L/2 \\ \rho L/2 \end{bmatrix}$$

where M is shown in the column form used in computation, since the off-diagonal terms are zero for the lumped mass matrix.

Note - as before a unit area is assumed.

The lumped mass procedure is utilized since it gives greater accuracy and fewer spurious oscillations than the corresponding consistent mass matrix in wave propagation problems.

The equation of state for defining the hydrostatic pressure (as before) is;

$$P^t = K \left[\frac{V_0}{V_t} - 1 \right]$$

For the 1-dimensional case with unit area;

$$P^t = K \left[\frac{L_0}{L_t} - 1 \right]$$

where K = bulk modulus, and P is the element pressure.

L_0, L_t = original length and length at time 't' respectively

$$L_0 = |x_2 - x_1|$$

$$L_t = |x_2^t - x_1^t|$$

A computer program for general one dimensional shock problems which incorporates the previous concepts is given at the end of this appendix. The temporal solution is obtained using the central difference operator which is described in Appendix A2.

A1.7.3 Verification Case

Consider a 1-D rigid pipe filled with H_2O travelling at an initial velocity $V_I = 100$ in/s:

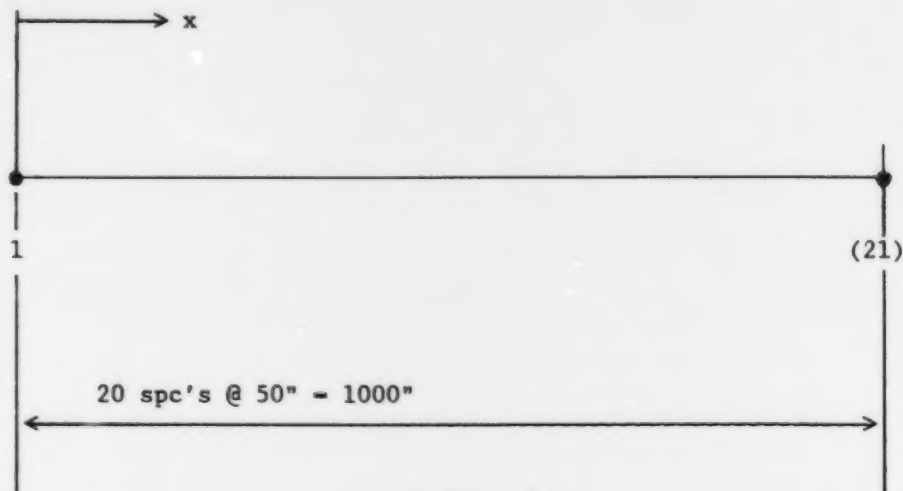


Figure A1.1

Total length	-	L_t	-	1000"
ρ	-			0.0361 lb _m /in ³
K	-	bulk modulus	-	300,000 psi

The velocity of the pressure wave:

$$(\text{acoustic velocity}) = C = \sqrt{386.4 \frac{K}{\rho}} = 56745 \text{ in/sec}$$

$$\begin{aligned} \text{Critical time step} = \Delta t_{cr} &= \frac{L^e}{C} = \frac{1000}{20 \times 56745} \\ &= 0.00088113 \text{ sec. (21 nodes)} \end{aligned}$$

$$M = \left\{ \begin{array}{c} 0.5L \\ L \\ L \\ \vdots \\ L \\ 0.5L \end{array} \right\} 9.345526E-5$$

where L is the element length.

$$F_{int}^t = \sum_{i=1}^{20} \left\{ \begin{array}{c} 1 \\ -1 \end{array} \right\}_i P_i^t$$

$$F_{ext}^t = \sum_{i=1}^{20} \left\{ \begin{array}{c} P_1^i(t) \\ P_2^i(t) \end{array} \right\}_i$$

The central difference operator (Appendix A2) is used as follows

$$\dot{M}u^t = F_{ext}^t + F_{int}^t$$

The time for a round trip pressure wave from node 1 to 21 and back is:

$$t_r = \frac{2L_t}{c} = 2 \cdot \left(\frac{1000}{56745} \right) = 0.03520 \text{ sec.}$$

The dynamic component of pressure P_D is:

$$P_D = \rho CV_I = \frac{0.036}{386.4} \times 56745 \times 100 = 529 \text{ psi}$$

$$\delta_v = \frac{L\Delta V}{V} = \frac{P_D L}{K} = \frac{529}{3000000} \times 1000 = 1.763''$$

at $t = 0.005287 \text{ sec.}$ the wave is located at

$$x = Ct = 300 \text{ in. from boundary (i.e. } x = 700 \text{ in.)}$$

As seen from figures A1.2 and A1.3, the shock wave propagation obtained from the 1-D code agrees closely with both the amplitude and shape of the analytical solution. In figure A1.4 excellent agreement is observed in the time history of displacement of the free surface.

A1.8 DISCRETIZATION IN 3-DIMENSIONAL SPACE

Following the derivation of the integral form of the equations given for the 1-D case, the three dimensional discretized equations are as follows;

$$M_e \dot{d}^e + F_{int}^e = F_{ext}^e$$

(on the element level)

The displacement/velocity fields are approximated by the interpolation of nodal variables using shape functions as follows;

$$\bar{u} \approx u = N d_x$$

$$\bar{v} \approx v = N d_y$$

$$\bar{w} \approx w = N d_z$$

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^k N_i \begin{Bmatrix} d_{xi} \\ d_{yi} \\ d_{zi} \end{Bmatrix} = \sum_{i=1}^k N_i d_i$$

where k is the number of nodes in the element, N_i is the shape function of node i.

The element considered is the eight noded linear isoparametric hexahedron element.

The element mass matrix is given as;

$$M_{ij}^e = \rho \int_V N_i N_j dV$$

Since a lumped mass formulation is used to take advantage of the efficiency of the central difference operator, the mass matrix, shown in column form, is:

$$M_i^e = \rho \frac{V}{8} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_i$$

For i = 1, 8 nodes

$$M^e \rightarrow (24 \times 1)$$

The element hydrostatic internal force vector F_{int}^e is given as;

$$F_{int}^e = - \int_V B^T P dv$$

$$B_i^T = \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix}$$

$$P = \frac{\sigma_x + \sigma_y + \sigma_z}{3} \quad ; \text{ At current time 't', node i}$$

The hydrostatic pressure P is constant within the element and,

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix} = J^{-1} \begin{bmatrix} \frac{\partial N_i}{\partial r} \\ \frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \end{bmatrix}$$

where r, s, t are the natural coordinates and the Jacobian J is;

$$J = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} & \frac{\partial z}{\partial t} \end{bmatrix}$$

For the 8 node linear isoparametric hexahedron shown in Figure A1.5, the shape functions, N_i , are given as:

$$N_i = \frac{1}{8}(1 + rr_i)(1 + ss_i)(1 + tt_i)$$

for natural coordinate system $r = s = t = \pm 1$

$$J_{11} = \frac{\partial x}{\partial r} = \sum_{i=1}^8 \frac{\partial N_i(r,s,t)}{\partial r} x_i$$

$$J_{12} = \frac{\partial y}{\partial r} = \sum_{i=1}^8 \frac{\partial N_i(r,s,t)}{\partial r} y_i$$

$$J_{13} = \frac{\partial z}{\partial r} = \sum_{i=1}^8 \frac{\partial N_i(r,s,t)}{\partial r} z_i$$

Similarly for J_{2i} , J_{3i}

Numerical Integration of F_{int}^e is carried out in the natural coordinate system (r, s, t) using a one point Gauss Quadrature rule: (i.e. at time 't')

$$t_{F_{int}}^e = -\int_V B^T P dv = - \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} B^T(r,s,t) P |J(r,s,t)| dr ds dt$$

using a one point quadrature:

$$t_{F_{int}}^e = -8 B^T(0,0,0) P |J(0,0,0)|$$

where $8 |J(0,0,0)|$ approximates the element volume, which is, in fact, exact for a parallelepiped.

The central difference operator is preferred for this problem which is governed by a rate constitutive relation, since the mid-point velocity gradient is required and this operator utilizes a mid-point formula.

The hydrostatic pressure P is determined from an equation of state relating volumetric strain to pressure as previously discussed.

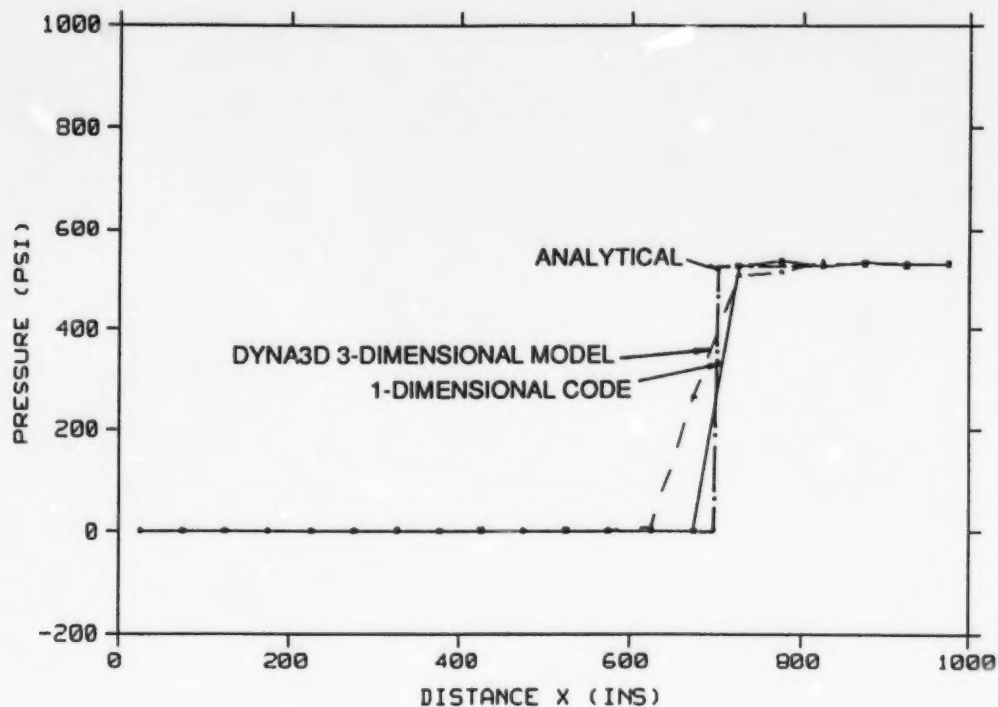


FIGURE A1.2

**FLUID FORMULATION TEST -
PRESSURE DISTRIBUTION TIME = 0.005287 s**

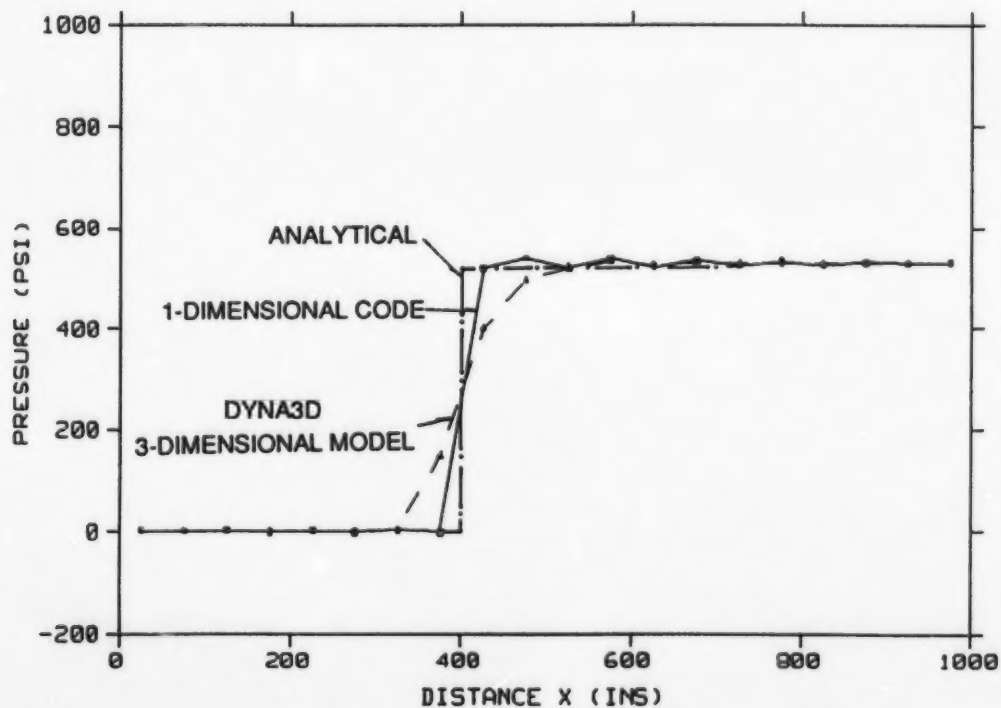


FIGURE A1.3

**FLUID FORMULATION TEST -
PRESSURE DISTRIBUTION TIME = 0.01057 s**

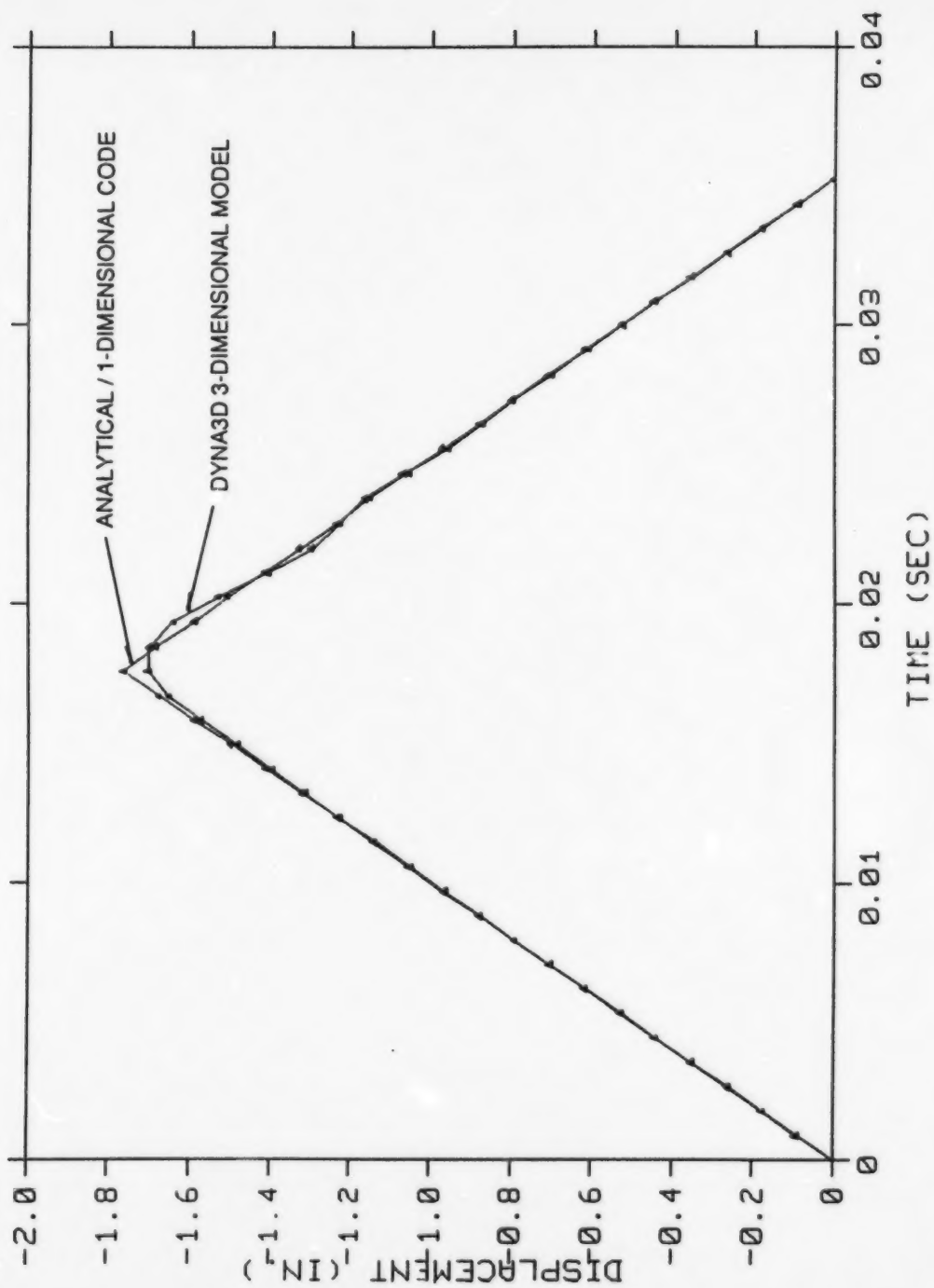


FIGURE A1.4

FLUID FORMULATION TEST -
DISPLACEMENT TIME HISTORY OF FREE SURFACE

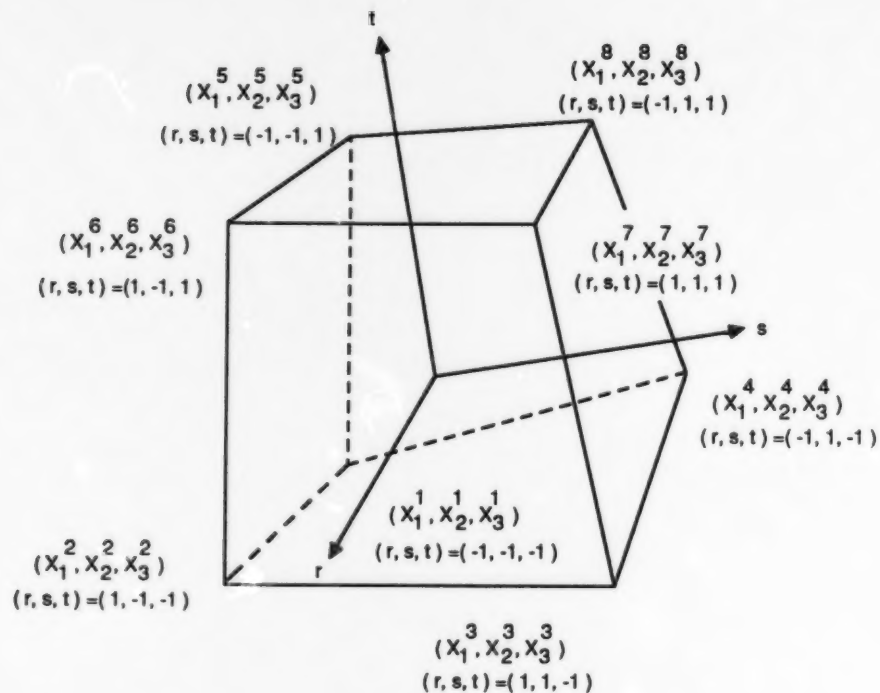
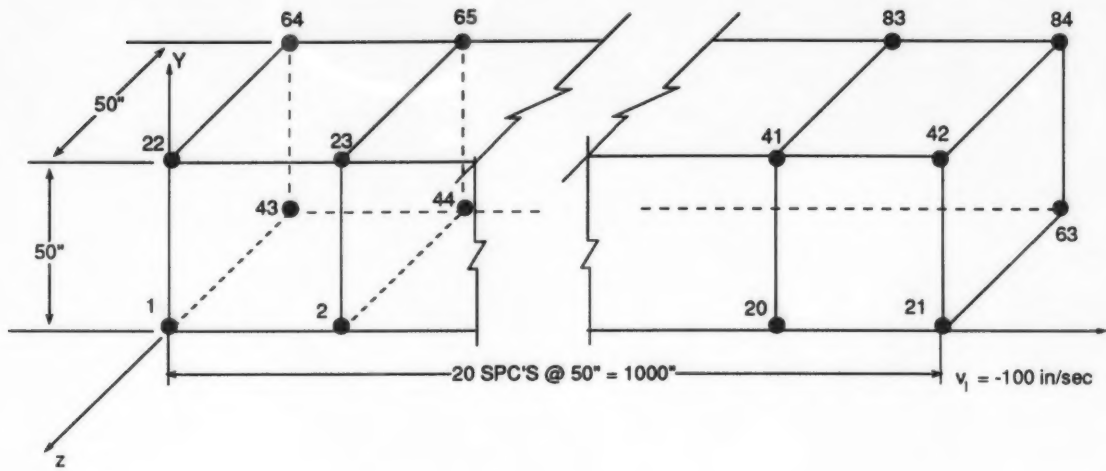
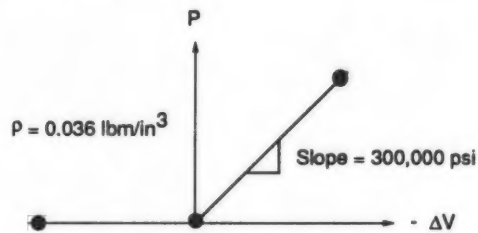


FIGURE A1.5
LINEAR ISOPARAMETRIC HEXAHEDRON ELEMENT

DYNA3D 1-D FLUID MODEL



EQ'N OF STATE: FOR (H₂O)



INITIAL VELOCITY OF FLUID COLUMN = -100 in/sec

FIGURE A1.6


```

C 1-DIMENSIONAL FLUID DYNAMIC MODEL TEST PROGRAM
C INVISCID FLUID , SIMPLIFIED E.O.S.
C 1-D WAVE PROPAGATION PROBLEM
C
C R. SAUVE , APRIL 1988
C-----
C DIMENSION AM(100),FINT(100),GL(100),NODE(2,100),X(100)
C DIMENSION X1(100),A1(100),V1(100),PRES(100),GLT(100),FEXT(100)
C
C IR=5
C IW=6
C IFL=10
C IPL1=11
C IPL2=12
C
C PRINT*,'READ: FLUID LENGTH,NO. OF NODES,INIT. VEL.,TIME SPAN, SCL'
C READ(IR,*)ALH,NN,VI,TEND,SCL
C WRITE(IFL,1101)ALH,NN,VI,TEND
1101 FORMAT(/,10X,'FLUID COLUMN LENGTH=',T40,F10.3,/,10X,
1 'NO. OF NODES=',T40,I5,/,10X,'INITIAL VELOCITY=',T40,F10.3,
1 /,10X,'SOLUTION TIME SPAN=',T40,F15.8,/)
C
C --INITIALIZATION--
C DO 595 J=1,NN
595 FEXT(J)=0.
C PRINT*,'READ: BNDRY PRESS,ACTIVATION TIME,ART. VISC (N=0,Y=1)'
C READ(IR,*)PB,TACT,IVISC
C NE=NN-1
C ELEMENT LENGTHS
C GL1=ALH/FLOAT(NE)
C DO 50 J=1,NE
C PRES(J)=0.0
C NODE(1,J)=J
C NODE(2,J)=J+1
50 GL(J)=GL1
C COORDINATES
C X1(1)=0.0
C DO 51 J=2,NN
51 X1(J)=X1(J-1) +GL1
C MATL. PROP'S
C RHO= 9.3455256E-5
C BULK=3.E05
C C=SQRT(BULK/RHO)
C MINIMUM CRITICAL TIME STEP
C DTMIN=SCL*GL1/C
C MASS MATRIX (LUMPED)
C AM(1)=RHO*GL1*0.5
C AM(NN)=RHO*GL1*0.5
C DO 52 J=2,NN-1
52 AM(J)=RHO*GL1
C INITIAL VELOCITIES , ELEMENT INTERNAL FORCES ,ORIGINAL COORDINATES
C DO 53 J=1,NN-1
C X(J)=X1(J)
C FINT(J)=0.0
53 V1(J)=VI
C
C TIME=0.0
C DT0=DTMIN
C DT=DTMIN
C E110=0.0
C

```

```

C  --SOLUTION (EXPLICIT CENTRAL DIFFERENCE OPERATOR)
C
C  CURRENT BOUNDARY PRESSURE
99  TSTRT=1.0
    IF(TACT.GT.0.0)TSTRT=TIME/TACT
    IF(TIME.LE.TACT)FEXT(1)=-FB*TSTRT
C  ACCELERATIONS
    DO 55 J=1,NN
      A1(J)= (-FEXT(J)+FINT(J))/AM(J)
C  CHECK BOUNDARY
      A1(NN)=0.0
    55 CONTINUE
      WRITE(IW,111)TIME,DT
      WRITE(IFL,111)TIME,DT
111  FORMAT(/,5X,'TIME= ',F15.8,5X,'TIME STEP=',F15.10)
      WRITE(IFL,102)
102  FORMAT(/,10X,'J',9X,'X1',8X,'V1',8X,'A1')
      DO 56 J=1,NN
        56 WRITE(IFL,100)J,X1(J),V1(J),A1(J)
100  FORMAT(6X,I5,3E11.3)
C
C  UPDATED COORDINATES, VELOCITIES
      DT2=(DT0+DT)*0.5
      DO 555 J=1,NN
        V1(J)=V1(J) +DT2*A1(J)
555  X1(J)=X1(J) +DT*V1(J)
C
      TIME=TIME +DT
      DT0=DT
      DT=1.E10
      IF(TIME.GT.TEND)STOP 'NORMALE'
C
C  --LOOP OVER ELEMENTS--
      DO 519 J=1,NN
        519 FINT(J)=0.0
        DO 520 J=1,NE
          C  DETERMINE ELEMENT PRESSURE FROM E. O. S.
          GLT(J)=(X1(NODE(2,J))-X1(NODE(1,J)))
          PRES(J)=BULK*(GL1/GLT(J)-1.0)
          C  WRITE(IW,1115)J,GLT(J),GL1,BULK,PRES(J),NODE(1,J),NODE(2,J)
          C 1115 FORMAT(5X,I4,4E15.5,2I5)
          C  BULK VISCOSITY
          PV=0.0
          Q=0.0
          IF(IVISC.LE.0)GO TO 320
          E11N=(GL1/GLT(J)-1.0)
          E11=(E11N-E110)/DT
          E110=E11N
          Q= (1.5*ABS(GLT(J)*E11)+ 0.06*C)
          PV=RHO*ABS(E11*GLT(J))*Q
          IF(E11.LT.0.)PV=0.0
          C
          320 FINT(NODE(1,J))=FINT(NODE(1,J)) -(PRES(J) +PV)
              FINT(NODE(2,J))=FINT(NODE(2,J)) +(PRES(J) +PV)
          C  FIND CRITICAL TIME STEP
          DTNEW=GLT(J)/(Q +SQRT(Q*Q+C*C))
          IF(DTNEW.LT.DT)DT= SCL*DTNEW
          520 CONTINUE
          C  STORE DISPL, VELOCITY AT NODE 1
              WRITE(IPL2,*)TIME,X1(1),V1(1)
          C  ELEMENT OUTPUT

```

```

      WRITE(IFL,111)TIME
      WRITE(IPL1,111)TIME
      DO 522 J=1,NE
      XPL=(X1(NODE(1,J))+X1(NODE(2,J)))*0.5
      WRITE(IPL1,1119)XPL,PRES(J)
1119  FORMAT(2F15.5)
      522 WRITE(IFL,1111)J,PRES(J),GLT(J)
1111  FORMAT(5X,I5,2F15.6)
C
C  -- END OF SOLUTION LOOP--
C  GO TO 99
C
      END

```

DYNA3D 3-DIMENSIONAL MODEL

```
** FLUID DYNAMIC TEST**
1,84,20,,,,,1,1,
0,0,0,0,0,0,1,
0.019,0.00086,0.00086,0,0,,,,.98
```

```
1,9,0.0000932,9
--HYDRODYNAMIC--
-100000.
```

```
-- EQN. OF STATE FOR H2O--
0.,-1.,
0.,300000.,
```

```
1,7,0.,0.,0.
2,5,50.0,0.,0.
21,5,1000.,0.,0.
22,7,0.,50.,0.
23,5,50.0,50.,0.
42,5,1000.,50.,0.
43,7,0.,0.,-50.
44,5,50.0,0.,-50.
63,5,1000.,0.,-50.
64,7,0.,50.,-50.
65,5,50.0,50.,-50.
84,5,1000.,50.,-50.
1,1,1,1,2,44,43,22,23,65,64
20,1,1,20,21,63,62,41,42,84,83
1,21
1,20
1,0.,0.,0.
2,-100.,0.,0.
21,-100.,0.,0.
22,0.,0.,0.
23,-100.,0.,0.
42,-100.,0.,0.
43,0.,0.,0.
44,-100.,0.,0.
63,-100.,0.,0.
64,0.,0.,0.
65,-100.,0.,0.
84,-100.,0.,0.
```

APPENDIX A2

CENTRAL DIFFERENCE OPERATOR



CENTRAL DIFFERENCE OPERATOR
NON-LINEAR STRUCTURAL DYNAMICS

A comparison of the routine used in a standard explicit central difference operator and the routine used in non-linear structural analysis follows.

STANDARD EXPLICIT CENTRAL DIFFERENCE OPERATOR

$$\dot{x}^t = \frac{1}{2\Delta t} [x^{t+\Delta t} - x^{t-\Delta t}] \quad (\text{velocity})$$

$$\ddot{x}^t = \frac{(x^{t+\Delta t} - x^t)}{\Delta t} - \frac{(x^t - x^{t-\Delta t})}{\Delta t} \quad (\text{acceleration})$$

$$\ddot{x}^t = \frac{1}{\Delta t^2} [x^{t+\Delta t} - 2x^t + x^{t-\Delta t}]$$

$$M\ddot{x}^t = p^t - F^t \quad (\text{damping not included})$$

where

$$F^t = f(x^t)$$

$$\frac{M}{\Delta t^2} [x^{t+\Delta t} - 2x^t + x^{t-\Delta t}] = p^t - F^t$$

$$\frac{M}{\Delta t^2} x^{t+\Delta t} = p^t - F^t + \frac{2M}{\Delta t^2} x^t - \frac{M}{\Delta t^2} x^{t-\Delta t} \quad (\text{displacement})$$

$$\dot{x}^t = \frac{1}{2\Delta t} [x^{t+\Delta t} - x^{t-\Delta t}] \quad (\text{velocity})$$

$$\ddot{x}^t = \frac{1}{\Delta t^2} [x^{t+\Delta t} - 2x^t + x^{t-\Delta t}] \quad (\text{acceleration})$$

$$\text{Initially } x^{-\Delta t} = x_0 - \dot{x}_0 \Delta t + \ddot{x}_0 \frac{\Delta t^2}{2}$$

where x_0 , $\dot{x}_0$, $\ddot{x}_0$ are the initial displacement, velocity and acceleration respectively.

From the velocity equation:

$$x^{t+\Delta t} = 2\Delta t \dot{x}^t + x^{t-\Delta t}$$

substituting into the equation for acceleration:

$$\Delta t^2 \ddot{x}^t = -2x^t + x^{t-\Delta t} + 2\Delta t \dot{x}^t + x^{t-\Delta t}$$

Thus;

$$x^{t-\Delta t} = x^t - \dot{x}^t \Delta t + \frac{\ddot{x}^t \Delta t^2}{2}$$

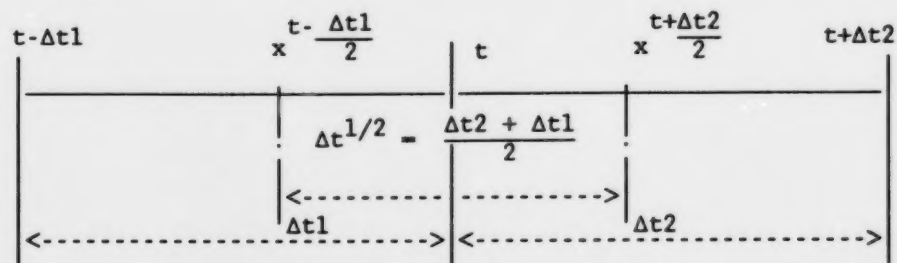
and

$$x^t = x_0 \text{ initially}$$

CENTRAL DIFFERENCE OPERATOR FOR NON-LINEAR F^t :

$$M\ddot{x}^t = P^t - F^t$$

where: $F^t = f(x^t)$



$$\bullet \ddot{x}^t = M^{-1}[P^t - F^t]$$

- $\Delta t^{1/2} = \frac{\Delta t_1 + \Delta t_2}{2}$
- $\dot{x} \frac{t+\Delta t_2}{2} = \dot{x} \frac{t-\Delta t_1}{2} + \Delta t^{1/2} \ddot{x}^t$
- $\dot{x} \frac{t+\Delta t_2}{2} = \frac{x^{t+\Delta t_2} - x^t}{\Delta t_2}$
- $x^{t+\Delta t_2} = x^t + \Delta t_2 \dot{x} \frac{t+\Delta t_2}{2}$
- update time t: $t + \Delta t_2$
- Obtain F^t and solve for $x^{t+\Delta t_2}$

Operator Stability

This operator is conditionally stable so that the time step ' Δt ' must be maintained at values below a critical time step Δt_{cr} (i.e., $\Delta t < \Delta t_{cr}$).

As before:

$$\ddot{x}^t = \frac{1}{\Delta t^2} \left[x^{t+\Delta t} - 2x^t + x^{t-\Delta t} \right]$$

Therefore when:

$$p^t = 0, F^t = Kx^t$$

$$x^{t+\Delta t} - 2x^t + x^{t-\Delta t} + \Delta t^2 M^{-1} Kx^t = 0$$

assuming a solution of a form;

$$x^{t+\Delta t} = \lambda(x^t)$$

$$x^t = \lambda(x^{t-\Delta t})$$

$$\lambda - 2 + 1/\lambda + \Delta t^2 M^{-1} K = 0$$

letting eigenvalues $w^2 = M^{-1}K$

$$\lambda^2 + \lambda(\Delta t^2 w^2 - 2) + 1 = 0$$

$$\lambda_{1,2} = \frac{-(\Delta t^2 w^2 - 2) \pm \sqrt{(\Delta t^2 w^2 - 2)^2 - 4}}{2}$$

The roots are complex if;

$$4 > (\Delta t^2 w^2 - 2)^2$$

Thus, in the limit;

$$\Delta t^2 w^2 - 2 = 2$$

$$\Delta t_{cr} \leq \frac{2}{w}$$

where, for structural problems (single degree of freedom system):

$$w_{max} = \left[|M^{-1}K^t|_{max} \right]^{1/2} = \left[\left| \frac{F^{t+\Delta t} - F^t}{M(x^{t+\Delta t} - x^t)} \right|_{max} \right]^{1/2}$$

$$\Delta t_{cr} \leq 2 \left[M \frac{(x^{t+\Delta t} - x^t)}{(F^{t+\Delta t} - F^t)} \right]^{1/2}$$

for shock wave problems:

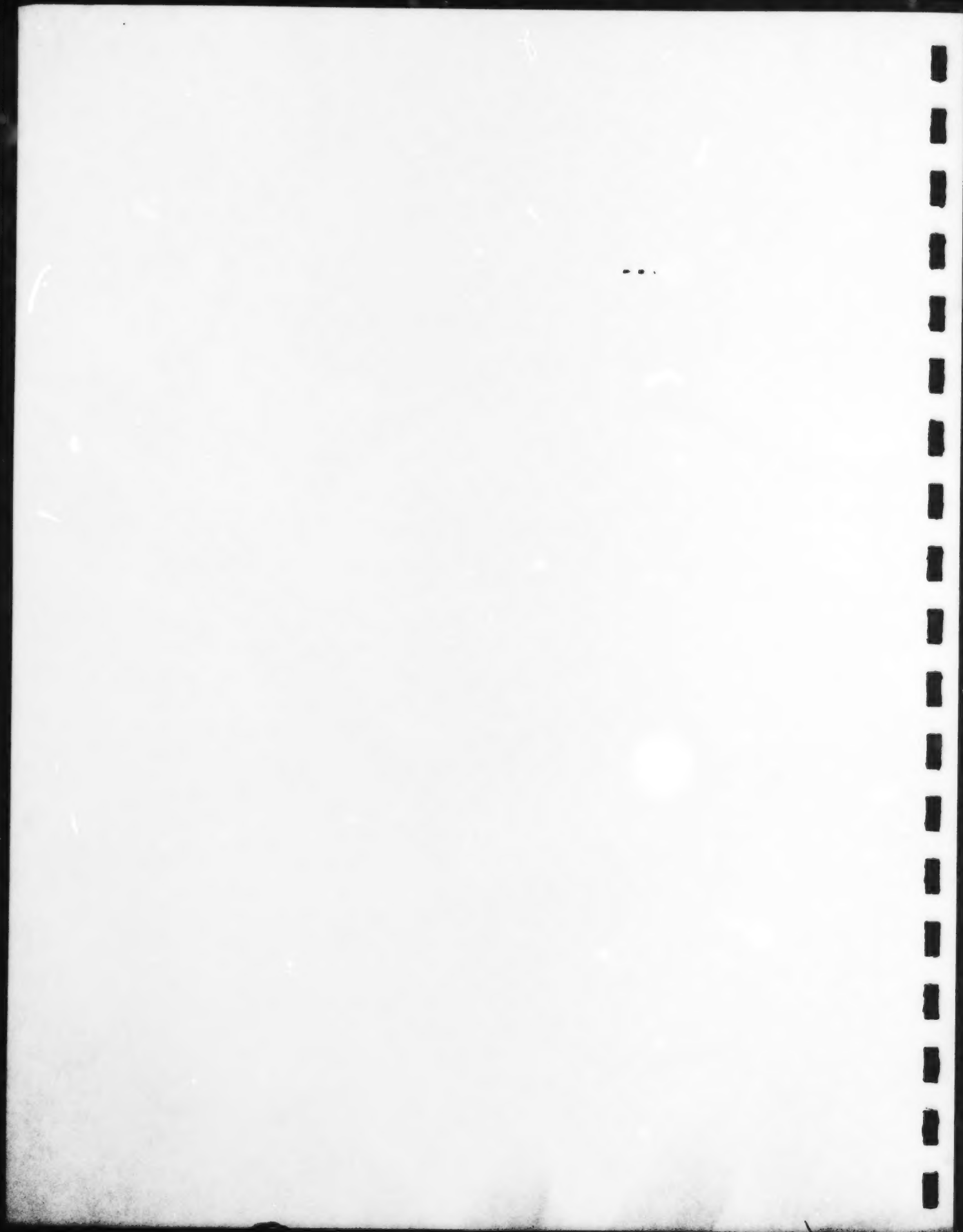
$$w_{max} = \frac{2C}{L}$$

C = speed of sound in the material

L = minimum element dimension

APPENDIX A3

DERIVATION OF FOAM MATERIAL CONSTANTS a_0 , a_1 , AND a_2 FOR
PRESSURE DEPENDENT FLOW RULE



DERIVATION OF MATERIAL CONSTANTS a_0 , a_1 , AND a_2 FOR
PRESSURE DEPENDENT FLOW RULE

Basic relations for foam material models based on elementary isotropic plasticity theory:

Total Stress = Hydrostatic Stress + Stress Deviator

$$\text{Hydrostatic Stress} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \sigma_p$$

where σ_i = principal stresses (positive in compression)

Decomposition of stress tensor:

$$\sigma_{ij} = \sigma'_{ij} + 1/3 \delta_{ij} \sigma_{kk}$$

where δ_{ij} = Krönecker delta

$$\therefore \sigma'_{ij} = \sigma_{ij} - \delta_{ij} \sigma_p$$

J_1 , J_2 , J_3 are the invariants of the deviatoric stress tensor

$$J_1 = (\sigma_x - \sigma_p + \sigma_y - \sigma_p + \sigma_z - \sigma_p) = 0$$

$$J_2 = \frac{1}{6} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)]$$

for pure tension/compression

$$J_2 = \frac{1}{6} [2\sigma_x^2] = \frac{\sigma_x^2}{3}$$

for pure shear

$$J_2 = \tau_{xy}^2$$

$$\text{Stress Tensor } \sigma_{ij} = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix}$$

Deviatoric Stress Tensor

$$\sigma'_{ij} = \begin{vmatrix} \sigma_x - \sigma_p & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma_p & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma_p \end{vmatrix}$$

J_2 can be obtained from the sum of the principal minors of σ'_{ij} .

In terms of the principal components

$$\sigma_p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$J_2 = \frac{1}{2} (\sigma_1'^2 + \sigma_2'^2 + \sigma_3'^2)$$

$$= \frac{1}{2} ((\sigma_1 - \sigma_p)^2 + (\sigma_2 - \sigma_p)^2 + (\sigma_3 - \sigma_p)^2)$$

for pure tension/compression $J_2 = \frac{\sigma_1^2}{3}$

For foams the yield function is given as (analogous to yield function for metals):

$$\phi = J_2 - (a_0 + a_1 \sigma_p + a_2 \sigma_p^2)$$

where σ_p is the hydrostatic pressure

$$\sigma_p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad \begin{matrix} (+ \text{ ve compression}) \\ (- \text{ ve tension}) \end{matrix}$$

The material is elastic when $\phi \leq 0$.

Tensile fracture occurs when

$$\sigma_p < \min \text{ root } [a_0 + a_1\sigma_p + a_2\sigma_p^2 = 0]$$

The yield function for foam has the form:

$$\phi = J_2 - (a_0 + a_1\sigma_p + a_2\sigma_p^2)$$

For uniaxial stress:

$$J_2 = a_0 + a_1\sigma_p + a_2\sigma_p^2 = \frac{1}{3} \sigma_y^2$$

where σ_y is the yield point in uniaxial compression corresponding to σ_p .

The yield surface for any stress state can be described by:

$$a_0 + a_1\sigma_p + a_2\sigma_p^2 = \frac{1}{3} \sigma_y^2$$

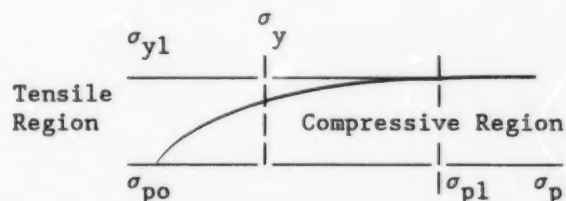
where σ_p is the hydrostatic pressure and σ_y is the equivalent uniaxial stress.

when $\sigma_p \leq \sigma_{po}$; $\sigma_{yo} = 0$

where σ_{po} is the tensile cutoff.

Parabolic Curve

$$\frac{\sigma_y^2}{3} = a_0 + a_1\sigma_p$$



$\sigma_p = \sigma_{po}$; $\sigma_y = 0$ at the tensile threshold,

Therefore:

$$a_0 = - a_1\sigma_{po}$$

at some other hydrostatic pressure σ_{p1} , the equivalent uniaxial stress at yield is given by σ_{y1} . Then

$$\frac{\sigma_{y1}^2}{3} = a_0 + a_1 \sigma_{p1} - a_1 \sigma_{p1} - a_1 \sigma_{po}$$

Therefore,

$$a_1 = \frac{\sigma_{y1}^2}{3(\sigma_{p1} - \sigma_{po})} ; a_0 = - \frac{\sigma_{po} \sigma_{y1}^2}{3(\sigma_{p1} - \sigma_{po})}$$

Conic

$$\sigma_y = C_1 + C_2 \sigma_p$$

$$\frac{\sigma_y^2}{3} = \frac{1}{3} [C_1^2 + 2C_1 C_2 \sigma_p + C_2^2 \sigma_p^2]$$

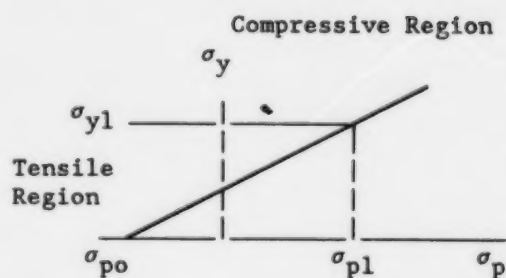
$$\sigma_p = \sigma_{po} ; \sigma_y = 0$$

$$C_1 = - C_2 \sigma_{po}$$

$$\sigma_{y1} = C_1 + \sigma_{p1} C_2 = - C_2 \sigma_{po} + \sigma_{p1} C_2$$

$$C_2 = \frac{\sigma_{y1}}{(\sigma_{p1} - \sigma_{po})}$$

$$C_1 = \frac{- \sigma_{po} \sigma_{y1}}{(\sigma_{p1} - \sigma_{po})}$$



$$\frac{1}{3} [C_1^2 + 2C_1 C_2 \sigma_p + C_2^2 \sigma_p^2] = a_0 + a_1 \sigma_p + a_2 \sigma_p^2$$

$$a_0 = \frac{(\sigma_{po}\sigma_{yl})^2}{3(\sigma_{pl} - \sigma_{po})^2} ; \quad a_1 = \frac{-2\sigma_{yl}^2 \sigma_{po}}{3(\sigma_{pl} - \sigma_{po})^2} ; \quad a_2 = \frac{\sigma_{yl}^2}{3(\sigma_{pl} - \sigma_{po})^2}$$

Thus, once values of the foam property characteristics are obtained from test, the flow rule can be derived in terms of constants a_0 , a_1 , and a_2 .

Calculation of a_0 , a_1 , and a_2 for 19 lb/ft³ Polyurethane Foam

The values of the constants used in the flow rule were evaluated using the test data obtained for the 19 lb/ft³ polyurethane foam, described in Ref 14.

A parabolic curve (i.e., $a_2 = 0$) was utilized, since, as pointed out in Ref 13, this fits the data for such materials very well.

It remains to calculate the constants a_0 and a_1 . From the test results, the tensile threshold or cut-off, that is, the pressure at which tensile fracture occurred, was found to be:

$$\sigma_{po} = -223.84 \text{ psi (negative denotes tension)}$$

Since the fracture occurs before yield:

$$\sigma_y = 0.$$

In addition to these values, a pure shear test was conducted and the yield in shear was found to be:

$$\tau_{yl} = 149.39 \text{ psi.}$$

The corresponding yield in tension is:

$$\sigma_{yl} = \sqrt{3} \tau_{yl} = 258.74 \text{ psi}$$

Since this is a case of pure shear:

$$\sigma_{pl} = 0.$$

Substituting these values into:

$$a_0 + a_1 \sigma_p = \frac{\sigma_y^2}{3} \quad (\text{parabolic curve, } a_2 = 0)$$

leads to the equality:

$$\begin{bmatrix} 1. & -223.84 \\ 1. & 0 \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{258.75^2}{3} \end{Bmatrix}$$

Solving for a_0 , a_1 :

$$\underline{a_0 = 22317.4}$$

$$\underline{a_1 = 99.7}$$

The final equation for the yield stress in terms of hydrostatic pressure is:

$$\underline{\sigma_y^2 = 3 (22317.4 + 99.7 \sigma_p)}$$

As implemented, the variation of σ_y with hydrostatic pressure σ_p is used for values of σ_p in compression only. When σ_p becomes tensile the value of yield, σ_y , stays constant at the value of σ_y corresponding to $\sigma_p = 0$. Once the tensile threshold is reached, the pressure is left at the cut-off value and the deviatoric stress components are zeroed.

The hydrostatic stress is governed by an equation of state relating pressure to volumetric strain whereas the pressure dependent flow rule governs the deviatoric behaviour.

As demonstrated in the experimental benchmarks performed in references 6, 7 and 8, the material model described above provides an acceptable representation of foam material. In fact, for the application considered, the foam material response is essentially governed by the equation of state.

APPENDIX A4
COMPUTER FILE RETRIEVAL



Listing of save set(s)

Save set: GREGFLUID.SAV
 Written by: GREG
 REC: [000501,000013]
 Date: 24-MAY-1988 13:02:03.72
 Command: BACKUP/SUFF=5/LOG [GREG.FLUID...]*.* MMD:GREGFLUID.SAV
 Operating system: VAX/VMS version V4.6
 BACKUP version: V4.6
 CPU ID register: 08000000
 Written on: MECH\$MMD:
 Block size: 8192
 Group size: 10
 Buffer count: 5

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[GREG.FLUID]A.FOR;2	3	3-MAY-1988	10:42
[GREG.FLUID]ALL.ELEM;6	174	11-MAY-1988	16:18
[GREG.FLUID]BOUND.FOR;5	3	9-MAY-1988	11:45
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 45 3-MAY-1988 11:51
 6 16-MAY-1988 10:14
 3 16-MAY-1988 10:24

Total of 172 files, 32341 blocks
 End of save set

APPENDIX A5
MODIFIED DENSITY CALCULATIONS

LID PORTION MODIFIED DENSITY CALCULATIONS

The THWTP weight distribution is as follows:

Overpack:	20,800 lbs.
Inner Container:	3,872 lbs.
Payload:	11,965 lbs.
Total THWTP weight:	36,637 lbs.

Since a bottom down flat end drop is considered, details of the overpack lid end of the package are approximated using a modified density.

Overpack Outer Shell:

$$\text{Volume} = 2\pi(48)^2 0.75 + \pi(48.75^2 - 48^2) (108) = 35477 \text{ in}^3$$

$$\text{Weight} = 0.29 \times 35477 = \underline{10,288 \text{ lbs.}}$$

Overpack Inner Shell:

$$\text{Volume} = 2\pi(36.5)^2 0.25 + \pi(36.75^2 - 36.5^2) (83) = 6867 \text{ in}^3$$

$$\text{Weight} = 0.29 \times 6867 = \underline{1,992 \text{ lbs.}}$$

Overpack Foam: (including 6.5 in. thickness of foam in lid portion)

$$\text{Volume} = 12\pi(48)^2 + \pi(48^2 - 36.5^2) 83 + \pi(48)^2 6.5 = 387293 \text{ in}^3$$

$$\text{Weight} = 387293 \times 0.011 = \underline{4260 \text{ lbs.}}$$

Remaining overpack weight to be distributed in lid portion:

$$\begin{aligned} &= 20800 - 10288 - 1992 - 4260 \\ &= 4260 \text{ lbs.} \end{aligned}$$

$$\text{Volume of Modified Lid} = \pi(48)^2 (6.5) = 47048 \text{ in}^3$$

$$\underline{\text{Modified Density}} = 4260 / 47048 = \underline{0.090 \text{ lb/in}^3}$$